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The Anatomy of Unemployment Dynamics

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Abstract

This paper examines the relation between individual unemployment durations and incidence on the one hand, and the time-varying macroeconomic conditions in the economy on the other. We allow for contemporaneous calendar time effects acting on the exit probabilities for all currently unemployed. Also, we allow for cohort effects on the exit probabilities by allowing the composition of the inflow into unemployment to depend on calendar time. In both cases we distinguish between business cycle effects and seasonal effects. The model is estimated with aggregate unemployment duration data, in which we allow for unobserved heterogeneity and correlated measurement errors. The results enable us to give a complete decomposition of the dynamics of unemployment over calendar time.

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1 Introduction

Unemployment has been a top issue for economic research and policy for many decades. Traditionally, microeconomic research focuses on the incidence and duration of unemployment on an individual level, while macroeconomic research focuses on the macro unemployment rate and its behavior over the business cycle. In the micro approach, attention has recently concentrated on dynamic duration models for explaining individual variation in unemployment duration. These models typically assume the parameters to be independent of macroeconomic conditions, and these conditions are at most included as an additional regressor. At the same time, the recently expanding macro literature on aggregate flows between labor market states stresses that the distribution of unemployment durations and incidence changes markedly over the business cycle. In the present paper we aim to bridge the gap between these approaches, by examining the relation between individual unemployment durations and incidence on the one hand, and the time-varying macroeconomic conditions in the economy on the other. In particular, we give a complete decomposition of the behavior of unemployment over calendar time, and we show how the shape of the individual exit probabilities out of unemployment varies over calendar time. The methodology we use is novel and generates striking implications that cannot be obtained using traditional approaches.

The most basic decomposition of aggregate unemployment is in terms of the gross size of the incidence and the average duration. Throughout the paper, we pay most attention to macroeconomic effects that can be identified from the unemployment duration distribution and the way it changes over calendar time. We present a model that allows individual exit probabilities out of unemployment, as functions of the elapsed unemployment duration, to depend on calendar time. This contemporaneous calendar time dependence is modeled as the product of seasonal effects and flexibly specified (yearly) business cycle effects. In addition to this, we allow for cohort effects, *i.e.* variation with the moment of inflow into unemployment, in a similar way. The estimates of the calendar time effects that are supposed to capture the business cycle effects can then be compared to the behavior of traditional economic business cycle indicators over time. We use observed individual characteristics to stratify the data, and we estimate models separately for different types of individuals. Thus, we can infer to what extent business cycle and seasonal effects differ between types, *i.e.* females and males.

As noted above, individual exit probabilities are allowed to depend on the elapsed duration of being unemployed. This represents genuine duration dependence due to *e.g.* stigma effects reducing the number of job opportunities of the long-term unemployed.¹ Apart from this, we also allow for unobserved heterogeneity in the exit probabilities. In the case of (unobserved) heterogeneity, individuals with the largest exit probabilities on average leave unemployment first. This dynamic sorting leads to a decline in the

¹See *e.g.* Vishwanath (1989) and Van den Berg (1990).

average quality of a cohort of unemployed in the course of time. Thus, negative duration dependence in observed aggregate exit probabilities may occur even in absence of genuine duration dependence at the micro level. This is important for policy analysis.²

The so called Mixed Proportional Hazard (MPH) assumption ensures identification of genuine and spurious duration dependence. The MPH model specifies the individual exit probability conditional on survival and the unobserved individual effect as the product of calendar time effects, the genuine duration dependence effect, and the unobserved individual effect (or ‘heterogeneity term’). As a result, the observed exit probabilities can be expressed as the product of calendar time effects, the genuine duration dependence effect, and the expected value of the heterogeneity term conditional on survival. It turns out that the latter is an interaction term of calendar time and elapsed duration. We extend earlier identification results by showing that such a model can be non-parametrically identified from aggregate unemployment duration data.

In the MPH model, the cohort effects can be interpreted as working by way of the composition of the individuals who flow into unemployment. For example, a relatively high exit probability for individuals entering unemployment in a certain season can be viewed as evidence that the inflow in this season contains relatively many individuals with high unobserved quality. Similarly, it can be investigated whether the composition of the inflow during a recession differs from the composition of the inflow at the top of a cycle. This provides a test of the model of Darby, Haltiwanger and Plant (1985), who argue that in a recession the inflow into unemployment contains a relatively large amount of individuals with small exit probabilities, and that this is the major cause of the procyclicality of observed exit probabilities from unemployment.

The model and estimation method developed in this paper are designed to be applicable to discrete-time time-series data on aggregate numbers of individuals in different unemployment duration classes. Such data can be used to calculate the aggregate outflow from different duration classes for each calendar time point. In our context, the main advantage of aggregate data is that they cover a much longer time span than is usual in micro data. Clearly, for reliable estimation of business cycle effects, it is necessary to have data that include at least a complete cycle. Another major advantage of aggregate data is that usually they do not suffer from attrition. In the analysis of labor market transitions, attrition is a particularly serious problem, since attrition out of panel survey data may be induced by the occurrence of a transition (see Van den Berg, Lindeboom and Ridder, 1994). Finally, truly aggregate data in principle cover the whole population, which makes such data better suited for the analysis of the overall impact of aggregate events like business cycles.

To date, a number of empirical studies have been published that focus on one or more of the issues we deal with in the present paper. Dynarski and Sheffrin (1990), Imbens

²See *e.g.* Layard, Nickell and Jackman (1991) and Van den Berg and Van Ours (1996).

and Lynch (1992) and Lollivier (1994) use micro data to estimate the effect of business cycle indicators like the unemployment rate on the unemployment duration distribution. Baker (1992a) uses aggregate data containing a large number of individual characteristics to investigate cyclical behavior of the determinants of unemployment. There have also been numerous studies on the relative importance of incidence and duration to explain variation in unemployment.³ Concerning the distinction of genuine duration dependence and unobserved heterogeneity we generalize the existing literature, which typically assumes functional form restrictions. Van den Berg and Van Ours (1994, 1996) provide a non-parametric analysis of duration dependence and heterogeneity in data similar to ours, but their statistical model has no clear stochastic foundation. Also, we correct for changes in the composition of the inflow when evaluating the interaction of calendar time and duration. Below we will compare our results to those in the literature. It should be noted from the outset that the vast majority of this empirical literature is based on U.S. data.

The outline of the paper is as follows. In Section 2 we discuss the model and the empirical implementation, and provide conditions under which the model is non-parametrically identified. Furthermore, we propose specification tests. Next, we estimate the model using data from France covering the 1980s and early 1990s. In Section 3 we present the data. Section 4 discusses the estimation results and the results of the specification tests. We illustrate the anatomy of unemployment dynamics with simulations. Section 5 concludes.

2 The model and the empirical implementation

2.1 Observation of unemployment

In this subsection we introduce measures of calendar time and unemployment duration. After that, we sketch the type of data we use, and we discuss the role of measurement errors. In the next subsection we then present the model for the exit probabilities out of unemployment.

We use two measures of time, each with a different origin. The variable t denotes the duration of a spell of unemployment for a given individual, as measured from the moment the individual becomes unemployed. The variable τ denotes calendar time, which has its origin somewhere in the past. We take t and τ to have the same measurement scale, apart from the difference in origin. Both t and τ are discrete variables. As an example, consider an individual who is unemployed for t periods at calendar time τ . If he fails to leave unemployment in period t , he will be unemployed for $t + 1$ periods at calendar time $\tau + 1$.

Ideally, aggregate data give the total numbers of individuals in the labor market who

³See *e.g.* the survey in Layard, Nickell and Jackman (1991).

are unemployed for t periods of time, $t = 0, 1, 2, \dots$, at calendar times $\tau_0, \tau_0 + 1, \tau_0 + 2$, etcetera. We denote these numbers by $U(t|\tau)$. From these numbers we can calculate the fraction $\theta(t|\tau)$ of the individuals who are unemployed for t periods at calendar time τ who leave unemployment at τ :

$$\theta(t|\tau) = \frac{U(t|\tau) - U(t+1|\tau+1)}{U(t|\tau)}. \quad (1)$$

This fraction equals the aggregate exit probability out of unemployment at calendar time τ and duration t , conditional on survival up to t . We take $U(0|\tau)$ as the measure of the size of the inflow into unemployment at calendar time τ .⁴ Note that aggregate unemployment at time τ is given by $U(\tau) := \sum_{t=0}^{\infty} U(t|\tau)$.

In reality we do not exactly observe the numbers $U(t|\tau)$, and therefore neither $\theta(t|\tau)$. Sometimes the data are based on surveys of unemployed individuals. Respondents may have trouble recalling their elapsed unemployment duration. In that case they may be counted as being unemployed for t periods of time whereas in reality they are unemployed for $t-1$ or $t+1$ periods. Alternatively, they may tend to round off their duration to the nearest natural unit of time, like an integer number of months. If only a sample of the population is surveyed then the data contain sampling errors as well. If the data cover the whole population and are based on administrative records, then there may be misclassifications due to administrative errors.

Because of this, we allow for measurement errors in the model. From now on we place a \sim on top of observed values of variables, in contrast to true or unobserved values. We assume that

$$\tilde{U}(t|\tau) = U(t|\tau) \varepsilon_{t,\tau} \quad (2)$$

with

$$\ln \varepsilon_{t,\tau} \sim N(0, \sigma^2).$$

In the empirical analysis we experiment with different types of correlation schemes for $\varepsilon_{t,\tau}$ over t and τ . If, at given calendar times τ , individuals are sometimes assigned to the wrong duration class adjacent to the right class, then we expect a negative correlation between $\varepsilon_{t,\tau}$ and $\varepsilon_{t+1,\tau}$ for every t . If the definition of unemployment used to count individuals at τ is less restrictive than the definition used elsewhere, then we expect a positive correlation between $\varepsilon_{t,\tau}$ and $\varepsilon_{t+1,\tau}$ for every t .

The observed exit probability out of unemployment $\tilde{\theta}(t|\tau)$ equals the right hand side of equation (1) with U replaced by \tilde{U} . By substituting equation (2) into this, we obtain

$$\ln(1 - \tilde{\theta}(t|\tau)) = \ln(1 - \theta(t|\tau)) + e_{t,\tau}, \quad (3)$$

⁴In Section 3 we return to the issue of measuring the size of the inflow.

where $e_{t,\tau} := \ln \varepsilon_{t+1,\tau+1} - \ln \varepsilon_{t,\tau}$. Thus, $e_{t,\tau} \sim N(0, 2(1 - \rho_{t,\tau})\sigma^2)$, where $\rho_{t,\tau}$ is the correlation between $\ln \varepsilon_{t,\tau}$ and $\ln \varepsilon_{t+1,\tau+1}$. Note that the errors in equation (3) are correlated even if the errors in equation (2) are mutually independent. In the latter case $\text{corr}(e_{t,\tau}, e_{t+1,\tau+1}) = -\frac{1}{2}$ for every t and τ , and all other types of correlations are zero.

Equation (3) links the data to the true exit probabilities. In the next subsection we present a model for these probabilities. Suppose we observe $\tilde{U}(t|\tau)$ for $K + 2$ duration classes $0, 1, \dots, K + 1$. Then (3) can be thought to represent $K + 1$ different equations: namely for $\tilde{\theta}(0|\tau)$ up to and including $\tilde{\theta}(K|\tau)$. The loss of information when going from $K + 2$ duration classes for U to $K + 1$ equations for θ , which is a first difference of U , concerns the level of unemployment. This is accounted for by the equation for the size of the total inflow into unemployment, *i.e.* $\tilde{U}(0|\tau) = U(0|\tau) \varepsilon_{0,\tau}$. In the next subsections we also present a simple model for $U(0|\tau)$.

2.2 The model

Usually, data sets on aggregate unemployment do not contain much information on individual characteristics that could be used as explanatory variables. At best the data are stratified into a small number of different types of individuals, in our case males and females. We estimate the model separately for each type, and in the sequel we present the model for a given type.

The aim is to provide a model for the true exit probabilities $\theta(t|\tau)$ appearing in the right hand side of equation (3). As stated in the introduction, we use a MPH model to describe these gross probabilities. The starting point for the MPH model is the specification of exit probabilities at the individual level. It is assumed that all variation in the individual exit probabilities out of unemployment can be explained by the prevailing unemployment duration t and calendar time τ and by unobserved heterogeneity across individuals. We denote the probability that an individual leaves unemployment right after t periods of unemployment, given that he is unemployed for t periods at calendar time τ , and conditional on his unobserved characteristics \mathbf{v} , by $\theta(t|\tau, \mathbf{v})$. We make the following assumptions on these individual (conditional) exit probabilities.

Assumption 1. (MPH) $\theta(t|\tau, \mathbf{v})$ has a mixed proportional hazard specification, *i.e.* there are positive functions ψ_1 and ψ_2 such that

$$\theta(t|\tau, \mathbf{v}) = \psi_1(t) \psi_2(\tau) \mathbf{v}. \quad (4)$$

For every t and τ , the distribution of \mathbf{v} conditional on calendar time τ and survival up to t is such that $\Pr(0 \leq \theta(t|\tau, \mathbf{v}) \leq 1) = 1$ and $\Pr(0 < \theta(t|\tau, \mathbf{v}) < 1) > 0$.

Assumption 2. Invariance of individual \mathbf{v} : \mathbf{v} does not change during unemployment.

The functions ψ_1 and ψ_2 represent the duration dependence and the calendar time dependence of the individual exit probabilities out of unemployment. Assumption 1 is

reminiscent of the standard MPH assumption in reduced-form models for micro duration data.⁵ In models for micro duration data, dependence on calendar time is usually ignored, and the role of τ in the model above is replaced by the role of observed explanatory variables x . An important difference between the present model and MPH models for micro data is that here we have discrete time, whereas in micro studies time is usually treated as continuous. The present model should not be interpreted as an approximation to the continuous time MPH model. Rather, it should be regarded as a flexible accounting device for discrete aggregate duration data, with an appealing interpretation. Because of the discrete time framework, we have to introduce the last line of Assumption 1. Note that this implies that $0 < \theta(t|\tau) < 1$, and that the support of \mathbf{v} is bounded, so that all moments of \mathbf{v} exist (see Subsection 2.5).

We now turn to the cohort effects. We assume these to act by way of the composition of the inflow, *i.e.* by way of the shape of the distribution of \mathbf{v} in the inflow. In particular, we allow a scale parameter of the distribution function $G_\tau(v)$ of \mathbf{v} in the inflow at calendar time τ to vary with τ , and we assume that this is the only way in which the distribution of \mathbf{v} varies between cohorts.⁶ Thus, we have

Assumption 3. The distribution $G_\tau(v)$ of unobserved heterogeneity at moment of inflow τ satisfies, for every τ ,

$$G_\tau(\psi_3(\tau)v) = G(v), \quad (5)$$

for some distribution function $G(v)$ that does not depend on τ , and some positive function ψ_3 .

If $\psi_3(\tau) > \psi_3(\tau')$ then the individuals entering unemployment at τ on average have higher values of their unobserved characteristics \mathbf{v} (*i.e.* higher exit probabilities) than individuals entering at τ' . It is intuitively clear that the class of functional forms for $\psi_3(\tau)$ has to be restricted to obtain an identifiable model. We turn to identification issues in the next subsection.

Assumptions 1–3 define a model for the aggregate exit probabilities $\theta(t|\tau)$, which, from a formal point of view, generalizes the standard MPH setup by allowing the unobserved heterogeneity distribution to depend on calendar time, which is our ‘observed explanatory variable’. To express the exit probabilities $\theta(t|\tau)$, appearing in the right hand side of equation (3), in terms of the individual exit probabilities $\theta(t|\tau, \mathbf{v})$, we have to integrate \mathbf{v} out of the latter. Let \mathbf{t} denote the random unemployment duration, and t its realization. In obvious notation, there holds that

$$\theta(t|\tau) \equiv \frac{\Pr(\mathbf{t} = t | \text{inflow at } \tau - t)}{\Pr(\mathbf{t} \geq t | \text{inflow at } \tau - t)} = \frac{\mathbb{E}_{\tau-t}(\Pr(\mathbf{t} = t | \text{inflow at } \tau - t; \mathbf{v}))}{\mathbb{E}_{\tau-t}(\Pr(\mathbf{t} \geq t | \text{inflow at } \tau - t; \mathbf{v}))}, \quad (6)$$

⁵See Lancaster (1990) for an extensive survey of such models.

⁶Here, v denotes a realization of the random variable \mathbf{v} .

in which the expectations $\mathbb{E}_{\tau-t}$ are taken with respect to the distribution $G_{\tau-t}(v)$. Using standard relations between probability density functions, hazards and survival functions (see *e.g.* Lancaster, 1990), the probabilities $\Pr(\mathbf{t} = t | \text{inflow at } \tau - t; \mathbf{v})$ and $\Pr(\mathbf{t} \geq t | \text{inflow at } \tau - t; \mathbf{v})$ can be expressed in terms of $\theta(t|\tau, \mathbf{v})$. By doing this, and substituting equations (4) and (5), we get

$$\theta(t|\tau) = \psi_1(t) \psi_2(\tau) \psi_3(\tau - t) \frac{\mathbb{E}(\mathbf{v} \prod_{i=1}^t (1 - \psi_1(t-i) \psi_2(\tau-i) \psi_3(\tau-t) \mathbf{v}))}{\mathbb{E}(\prod_{i=1}^t (1 - \psi_1(t-i) \psi_2(\tau-i) \psi_3(\tau-t) \mathbf{v}))}, \quad (7)$$

in which we use the convention that the product term is one if $t = 0$. The expectations are now taken with respect to the distribution G (see Assumption 3). Substitution of equation (7) into equation (3) establishes the link between the observed exit probabilities and the model determinants.⁷

Our model is closed by the specification of an equation for the inflow size, the incidence equation. We simply take

$$U(0|\tau) = \psi_4(\tau), \quad (8)$$

for some positive function ψ_4 . Substitution of (8) into equation (2) for $t = 0$ establishes the link between the observed $\tilde{U}(0|\tau)$ and the unknown function $\psi_4(\tau)$.

2.3 Identification

The structural determinants in our model are the functions $\psi_1, \psi_2, \psi_3, \psi_4$ and G . As ψ_4 is trivially identified from the $\tilde{U}(0|\tau)$ data, we can restrict attention to identification of ψ_1, ψ_2, ψ_3 and G from the duration data.

Suppose we consider the duration model (7) for durations $0, 1, \dots, K$ and calendar time periods $\tau_0, \tau_0 + 1, \dots, \tau_0 + N$, with $K, N \geq 0$ and $K \leq N$, where possibly $N = \infty$ or $K = N = \infty$. It is convenient to exclude (τ, t) for which (7) involves $\psi_3(\tau - t)$ for $\tau - t < \tau_0$, *i.e.* to restrict attention to cohorts flowing into unemployment within the given calendar time frame. Formally, let $D_K = \{0, 1, \dots, K\}$ and $T_N = \{\tau_0, \tau_0 + 1, \dots, \tau_0 + N\}$ denote the sets of duration classes and calendar time periods considered. Furthermore, let $T_N(t) := \{\tau \in T_N | \tau - t \geq \tau_0\}$ contain all calendar time moments for which duration class t is considered, and let $\mathcal{T}_{N,K}(k) := \{(\tau, t) \in D_K \times T_N | t \leq k \wedge \tau \in T_N(t)\}$ combine all such pairs (τ, t) up to and including duration class k . We will discuss identification of our model for $(\tau, t) \in \mathcal{T}_{N,K} := \mathcal{T}_{N,K}(K)$.

⁷The model developed so far is similar to the model analyzed in Van den Berg and Van Ours (1996) and Van den Berg and Van Ours (1994). Those papers focus entirely on the issue of distinguishing genuine duration dependence from unobserved heterogeneity. A simple empirical strategy is developed and applied that enables the estimation of ψ_1 and G from equations in which ψ_2 has cancelled out and ψ_3 is seasonal. Contrary to the present model, the model in those papers lacks a clear stochastic foundation and does not address business cycle issues.

Let $\mu_i := \mathbf{E}(\mathbf{v}^i)$ denote the moments associated with G . By expanding the product terms in equation (7) we find that $\theta(t|\tau)$ depends on $\{\psi_1(i), \psi_2(\tau - t + i), \psi_3(\tau - t), \mu_{i+1}, \text{ with } i = 0, 1, \dots, t\}$. Thus, the duration model (7) on $\mathcal{T}_{N,K}$ can be represented by a ‘parameter’ vector $\psi := (\psi_1, \psi_2, \psi_3, \mu_1, \gamma)$, where

$$\begin{aligned}\psi_1 &:= (\psi_1(0), \dots, \psi_1(K)), \\ \psi_2 &:= (\psi_2(\tau_0), \dots, \psi_2(\tau_0 + N)), \\ \psi_3 &:= (\psi_3(\tau_0), \dots, \psi_3(\tau_0 + N)), \\ \gamma &:= (\gamma_2, \dots, \gamma_{K+1}),\end{aligned}$$

and $\gamma_i := \mu_i/\mu_1^i$.⁸ Now, let $\theta(t|\tau)$ and $\hat{\theta}(t|\tau)$ equal the r.h.s. of (7) evaluated at ψ and $\hat{\psi}$, respectively.

Definition 1. Two parameterizations ψ and $\hat{\psi}$ are *observationally equivalent* on $\mathcal{T}_{N,K}$ if $\theta(\tau, t) = \hat{\theta}(\tau, t)$ for all $(\tau, t) \in \mathcal{T}_{N,K}$.

We discuss identification of our model in terms of the set of observationally equivalent parameterizations. We restrict this set by three assumptions.

First, we assume that calendar time variation is separable in seasonal variation and variation over years covering full seasonal cycles. Let the number of seasons $S \geq 2$ be given, and label seasons by elements of $\mathcal{S} := \{1, 2, \dots, S\}$. Supposing that τ_0 corresponds to the first season, the number of years equals $[N/S] + 1$, so that we can index the years by elements of $Y_N := \{0, 1, \dots, [N/S]\}$.⁹ Let $y : T_N \rightarrow Y_N$ map calendar time τ into years y : $y(\tau) = [(\tau - \tau_0)/S]$. Also, let $s : T_N \rightarrow \mathcal{S}$ add a season s to each calendar time moment τ : $s(\tau) = 1 + \tau - \tau_0 - Sy(\tau)$. Then, we have

Assumption 4. $\ln \psi_2$ and $\ln \psi_3$ are additively separable in seasonal and yearly terms:

$$\begin{aligned}\ln \psi_2(\tau) &= \omega_2(s(\tau)) + \alpha_2(y(\tau)) \quad \text{and} \\ \ln \psi_3(\tau) &= \omega_3(s(\tau)) + \alpha_3(y(\tau)),\end{aligned}$$

where ω_2 and ω_3 are functions only depending on the prevailing season, and α_2 and α_3 functions only depending on the prevailing year.

Thus, we assume that all calendar time variation at higher than yearly frequencies can be captured by seasonal effects that are repetitive between years. The lower frequency effects are modeled by ‘year dummies’, where the first season of each year is always an integer number of full seasonal cycles away from τ_0 .

Second, as our model includes both cohort $(\tau - t)$, contemporaneous calendar time (τ) and duration (t) effects, two parameterizations $(\psi_1, \psi_2, \psi_3, \mu_1, \gamma)$ and $(f^t \psi_1, f^{-\tau} \psi_2, f^{\tau-t} \psi_3, \mu_1, \gamma)$

⁸Note that the number of ‘parameters’ grows with both N and K , and is infinitely large for $N = \infty$. As such, the analysis is fully non-parametric. See also the discussion on estimation in the next subsection.

⁹Here $[\cdot]$ denotes the entier function.

are observationally equivalent for any positive constant f . Therefore, we exclude an exponential trend from ψ_3 by^{10,11}

Assumption 5. The log yearly cohort effect is orthogonal to a linear trend on Y_N :

$$\sum_{y \in Y_N} (2y - [N/S]) \alpha_3(y) = 0. \quad (9)$$

Clearly, if some $\alpha_3(y)$ is orthogonal to a linear trend in this sense, then, for any constant $g \neq 0$, $\alpha_3(y) + gy$ is not orthogonal to a trend. On the other hand, if $\alpha_3(y)$ is not orthogonal to a trend, we can find a constant g such that $\alpha_3(y) + gy$ is. Thus, Assumption 5 effectively restricts the scope for reassigning an exponential trend from $\psi_3(\tau - t)$ on the one hand to $\psi_1(t)$ and $\psi_2(\tau)$ on the other, without changing the image of $\psi_1(t)\psi_2(\tau)\psi_3(\tau - t)$.¹²

Third, in order to identify the unobserved heterogeneity distribution, we need variation in the ‘regressor effects’, ψ_2 and ψ_3 :

Assumption 6.

$$\exists \tau, \tau' \in T_N(K) : \psi_2(\tau - t)\psi_3(\tau - K) > \psi_2(\tau' - t)\psi_3(\tau' - K) \text{ for all } t \in D_K.$$

Assumption 6 requires the existence of two cohorts of unemployed, flowing in at $\tau - K$ and $\tau' - K$ respectively, such that one cohort has higher mean exit probabilities in each duration class in D_K . The role of this assumption is standard. Unobserved heterogeneity induces negative duration dependence at the aggregate level: the fourth factor in the r.h.s. of (7), the conditional expectation of the unobserved heterogeneity term, decreases with t . If exit probabilities were constant over calendar time, this negative duration dependence would be the same for all τ , and indistinguishable from duration dependence at the individual level, ψ_1 . Assumption 6 ensures that observed duration dependence differs between the cohorts flowing in at $\tau - K$ and $\tau' - K$. Then, G , or, more precisely, γ can be identified from this difference, or from the interaction of τ and t in $\theta(t|\tau)$.¹³ Assumption 6 is not very restrictive for small enough K , typically not larger than half the length of a

¹⁰For practical purposes it is convenient to restrict the orthogonality requirement to the year effects. Seasonal cycles are generally not orthogonal to a trend for finite N , but the ‘yearly trend’ embodied in a seasonal cycle is limited by its repetitive character and vanishes as $N \rightarrow \infty$.

¹¹The trend in (9) is chosen to be orthogonal to a constant (*i.e.*, to average 0 over Y_N), so that we do not confuse orthogonality to a trend with any normalization of α_3 in Assumption 5.

¹²Similar assumptions are used in the consumption literature to identify time, cohort and age effects. See Deaton and Paxson (1993,1994).

¹³Van den Berg and Van Ours (1996) and Abbring, Van den Berg and Van Ours (1997) discuss this issue in more detail. There is an analogy with MPH models for micro duration data, in which the role of τ is replaced by observed regressors. Elbers and Ridder (1982) provided the first identification proof using variation in observed regressors, and Van den Berg (1992) and Melino and Sueyoshi (1990) discuss the role of interaction of these regressors with duration dependence. The main difference with our model is that in these standard micro MPH models the regressors are assumed to be constant over the duration of the spells, so that (6) reduces to a simpler ‘static’ condition.

business cycle. It should be noted that Assumption 6 is not necessary for identification. In particular, if K is large Assumption 6 will not hold, but more relaxed conditions can be applied. However, these conditions are not as transparent as Assumption 6. Therefore, and because our empirical analysis is based on a small number of duration classes only, we will not pursue this any further.¹⁴

Let Ψ be the set of all parameter vectors ψ that satisfy Assumptions 1–6. In the Appendix we prove the following proposition.

Proposition 1. *If two parameterizations $\psi, \hat{\psi} \in \Psi$ are observationally equivalent on $\mathcal{T}_{N,K}$, then $\hat{\psi}_1(t) = a\psi_1(t)$ for all $t \in D_K$, $\hat{\alpha}_2(\tau) = \alpha_2(\tau) + \ln b$, $\hat{\omega}_2(\tau) = \omega_2(\tau) + \ln c$, $\hat{\alpha}_3(\tau) = \alpha_3(\tau) + \ln d$ and $\hat{\omega}_3(\tau) = \omega_3(\tau) + \ln e$ for all $\tau \in T_N$, and $\hat{\mu}_1 = (abcde)^{-1}\mu_1$ and $\hat{\gamma} = \gamma$, for some positive constants a, b, c, d and e .*

Proposition 1 implies that, under Assumptions 1–6, we can identify ψ up to 5 unknown parameters, a, b, c, d and e . These parameters redistribute the overall scale of the exit probabilities between the 6 factors in our multiplicative probability model, and can be pinned down by innocuous normalizations. Note that we can only identify the moments of the unobserved heterogeneity distribution G . However, Assumption 1 implies that G has bounded support. Then, if we know $\{\mu_i\}_{i=1}^\infty$, *i.e.* if $K = \infty$, we can uniquely determine G . In practice $K < \infty$, and $\{\mu_i\}_{i=1}^{K+1}$ only provides bounds on G . We return to this in Subsection 2.5.

A final remark is that there will always be alternative models that are observationally equivalent to our model. A trivial example is the non-MPH model in which *individual* exit probabilities are given by equation (7) and in which there is no unobserved heterogeneity. Another example is a model in which the composition of the inflow is constant over time, and in which the individual exit probabilities $\theta(t|\tau, \mathbf{v})$ depend on the moment of inflow $\tau - t$ by way of a multiplicative factor $\psi_3(\tau - t)$. In that case the distribution of characteristics in the inflow does not change, but becoming unemployed at certain dates gives the exit probabilities a boost. De Toldi, Gouriéroux and Monfort (1992) take this approach to model the effect of the season at the moment of inflow.

2.4 Empirical implementation and estimation

For expositional purposes, we assume that calendar time variation of the incidence can also be separated in seasonal and yearly effects, or

$$\ln \psi_4(\tau) = \ln(\nu_1) + \omega_4(s(\tau)) + \alpha_4(y(\tau)),$$

where ω_4 and α_4 are again functions only depending on the prevailing season and year,

¹⁴See also the footnote to the Appendix.

respectively, and ν_1 is a positive constant.¹⁵ Note that, without loss of generality, we can write α_2 and α_4 as the sum of a cyclical component and a linear trend in the sense of Assumption 5. Denote the cyclical part of α_k by α_k^\perp , $k = 2, 3, 4$. Then, the linear trend term in α_k is given by $\alpha_k - \alpha_k^\perp$, and can be represented by the yearly change of the trend, or $\Delta(\alpha_k(y) - \alpha_k^\perp(y))$, $k = 2, 3, 4$. Clearly, this yearly change is independent of y , so that we can simply write $\Delta(\alpha_k - \alpha_k^\perp)$. Obviously, $\Delta(\alpha_3 - \alpha_3^\perp) = 0$, and even $\alpha_3 - \alpha_3^\perp = 0$, by Assumption 5. Using this notation, we can fully characterize the duration model by

$$\begin{aligned} \mu_1, \gamma_2, \gamma_3, \dots, \gamma_{K+1}; & \quad \ln \psi_1(0), \ln \psi_1(1), \dots, \ln \psi_1(K); \\ \omega_2(1), \omega_2(2), \dots, \omega_2(S); & \quad \alpha_2^\perp(1), \alpha_2^\perp(2), \dots, \alpha_2^\perp([N/S]); \quad \Delta(\alpha_2 - \alpha_2^\perp) \\ \omega_3(1), \omega_3(2), \dots, \omega_3(S); & \quad \alpha_3^\perp(1), \alpha_3^\perp(2), \dots, \alpha_3^\perp([N/S]), \end{aligned}$$

and the incidence model by

$$\begin{aligned} \nu_1; \\ \omega_4(1), \omega_4(2), \dots, \omega_4(S); & \quad \alpha_4^\perp(1), \alpha_4^\perp(2), \dots, \alpha_4^\perp([(N+1)/S]); \quad \Delta(\alpha_4 - \alpha_4^\perp), \end{aligned}$$

and, in each case, the disturbance parameters. In the remainder of the paper we will present results in terms of these quantities, which we will call the ‘parameters’ of the model.

Following the previous subsection, we ensure identification of the parameters by appropriate normalizations and orthogonality restrictions. We normalize $\ln \psi_1(0) = 0$, $\omega_2(1) = 0$, $\omega_3(1) = 0$, $\omega_4(1) = 0$, and the average values of $\alpha_2^\perp(y)$, $\alpha_3^\perp(y)$ and $\alpha_4^\perp(y)$ over Y_N , Y_N and Y_{N+1} , respectively, to 0. Furthermore, $\alpha_2^\perp(y)$, $\alpha_3^\perp(y)$ and $\alpha_4^\perp(y)$ are forced to be orthogonal to a linear trend over Y_N , Y_N and Y_{N+1} , respectively.

We do not impose any additional restrictions, and aim at a fully non-parametric analysis. In particular, we do not specify the α_k^\perp as regressions on business cycle indicators, nor do we assume that the business cycles in the α_i are regular or periodical in any sense. Instead, we assess the relation with the general business cycle by comparing the estimation results to the way in which traditional business cycle indicators behave over time.¹⁶ Drawback of this approach is that our model has incidental parameters in both the calendar time and the duration dimensions.¹⁷ For convenience, and by lack of suitable

¹⁵As we model $U(0|\tau)$ on T_{N+1} , and not just T_N , s and y should now be extended to add seasons in \mathcal{S} and years in Y_{N+1} to calendar time in T_{N+1} . However, the specifications of s and y in Subsection 2.3 apply without change. The constant is only added to harmonize normalizations of the duration and incidence models; see below. Note that, unlike Assumption 4, this assumption is not necessary for identification.

¹⁶Thus, we also avoid circularities possibly arising from regressing aggregate exit probabilities on business cycle indicators closely related to unemployment duration itself. See the discussion on the ‘reflection problem’ by Manski (1993).

¹⁷The dimensionality of the α_i^\perp functions can be reduced by approximating the year effects by series of orthogonal polynomials (Abbring, Van den Berg and Van Ours, 1994). However, as we would typically allow the order of the polynomials to increase with N , this does not solve the incidental parameter problem.

alternatives, we opt for maximum likelihood (ML) estimation. As we should expect standard asymptotic theory on the ML estimator to fail, we have performed some Monte Carlo simulations to assess its finite sample properties. Although we find some evidence for biases, deviations from the true parameter values are always small compared to both the Monte Carlo and the asymptotic ML estimates of the standard errors of the ML estimator. From this we conclude that ML provides acceptable estimates of the parameters of our model.¹⁸

2.5 Specification tests

As is clear from Subsection 2.3, the estimates of the duration dependence function ψ_1 and the unobserved heterogeneity distribution G crucially depend on the MPH specification. Thus, it is particularly important to test for this. In this subsection we develop tests that are based on the estimates of μ_1 and γ .

The last line of Assumption 1 implies that $0 \leq \mathbf{v} \leq 1/(\psi_1(t)\psi_2(\tau))$ $G_{\tau-t}$ -almost surely for all t and τ . By using equation (5) it can be seen that this is equivalent to the requirement that

$$0 \leq \mathbf{v} \leq \bar{v} \quad G - \text{a.s.}, \quad \text{where } \bar{v} = \inf_{(\tau, t) \in \mathcal{T}_{N, K}} \left(\frac{1}{\psi_1(t)\psi_2(\tau)\psi_3(\tau - t)} \right). \quad (10)$$

Thus, the support of G is bounded from below by zero, and from above by $\bar{v} < \infty$. It can be shown that the sequence of moments $\{\mu_i\}$ corresponds to a distribution G satisfying this requirement if and only if $\{\bar{v}^{-i}\mu_i\}$ is *completely monotone*, *i.e.*

$$(-1)^j \Delta^j \bar{v}^{-i} \mu_i \geq 0 \text{ for } j = 0, 1, \dots, i \text{ and } i = 0, 1, \dots, K + 1, \quad (11)$$

where $\mu_0 = 1$. If $K = \infty$, G is uniquely determined. From a finite sequence of moment conditions bounds on G can be constructed.¹⁹ Note that the additional requirement that $\Pr(0 < \theta(t|\tau, \mathbf{v}) < 1) > 0$ puts mild further restrictions on the moment sequence, like $0 < \mu_1 < 1$.

Since complete monotonicity is not imposed on the moments while estimating the model, we can test for it.²⁰ In general, however, the test statistics implied by the inequalities are not very appealing. First, the upper bound equals the infimum of a number of

¹⁸We also find that Monte Carlo estimates of the standard errors of the estimator are generally somewhat higher than the asymptotic ML estimates corresponding to the Hessian representation of the information matrix. It should be noted that any such deviations from predictions from standard asymptotic theory can both be due to differences between finite sample and asymptotic properties of the ML estimator and to the incidental parameter problem. In the sequel, we simply report the ML estimates.

¹⁹This follows from a simple change of variables in the Hausdorff moment problem, which is concerned with distributions concentrated on $[0, 1]$. See Shohat and Tamarkin (1943) and Widder (1946).

²⁰If we do not require G to be a distribution function on $[0, \bar{v}]$, but allow G to be any function of bounded variation, the moments can take any value on the real line, and equation (7) specifies a more general class of models for the exit probabilities. For $0 < \theta(t|\tau) < 1$ only a subset of the conditions

different functions of parameters, so the asymptotic distribution of the statistics is not normal and does not have a well-known shape. Moreover, they do not depend only on the μ_i estimates but also on the estimates of the other parameters of the model. For these reasons we do not construct tests from the complete monotonicity requirement directly.

Instead, we first focus on the lower bound of zero only. A necessary condition for the existence of a distribution with nonnegative support with moments $\{\mu_i\}_{i=1}^{K+1}$ is that the moment matrices $(\mu_{i+j})_{i,j=0\dots k_e}$ and $(\mu_{i+j+1})_{i,j=0\dots k_o}$ are positive semi-definite, for $k_e = 0, 1, \dots, [(K+1)/2]$ and $k_o = 0, 1, \dots, [K/2]$.²¹ For $K = 4$, this puts constraints on the following functions of γ :

$$\begin{aligned} q_2(\gamma) &= \gamma_2 - 1, \\ q_3(\gamma) &= \gamma_3 - \gamma_2^2, \\ q_4(\gamma) &= \gamma_2\gamma_4 - \gamma_3^2 - \gamma_4 - \gamma_2^3 + 2\gamma_2\gamma_3, \text{ and} \\ q_5(\gamma) &= \gamma_3\gamma_5 - \gamma_4^2 - \gamma_2^2\gamma_5 - \gamma_3^3 + 2\gamma_2\gamma_3\gamma_4. \end{aligned} \tag{12}$$

If $q_i(\gamma) < 0$ for some i then no distribution with positive support is able to generate these γ as normalized moments.²² If $q_i(\gamma) \geq 0$ for all i , then there are such distributions, except for some boundary cases, like $\gamma_2 = 1$ and $\gamma_3 > 1$. So, a relatively simple procedure is to test null hypotheses $q_i(\gamma) \geq 0$ by using the corresponding estimates.

We also check a number of the remaining conditions, which are implied by the parameter restrictions that follow from the finite upper bound. For example, from equation (7) it is clear that $\bar{v} \leq \theta(0|\tau)/\mu_1$, so that (11) should at least hold for an upper bound of $\theta(0|\tau)/\mu_1$. Substituting in (11) and evaluating for $j = 1$ gives $\theta(0|\tau) \leq \gamma_2/\gamma_3$, $\theta(0|\tau) \leq \gamma_3/\gamma_4$, etcetera.²³ By comparing the estimates of the right-hand sides of these inequalities to the observed $\hat{\theta}(0|\tau)$, and taking account of standard errors, one can get a feeling on whether these inequalities hold.

The moment tests proposed above are informative on the validity of Assumption 1. Suppose that in reality $\theta(t|\tau, \mathbf{v})$ is not multiplicative in t , τ and \mathbf{v} , but instead contains interaction terms. Then, in particular cases, this shows up in the γ estimates being inconsistent with the moment restrictions above. For example, suppose that the duration dependence pattern for individuals with large v differs from that for individuals with small

introduced in this subsection is required. Furthermore, the proof of Proposition 1 only relies on this subset of conditions. Thus, there exist moment sequences that correspond to $0 < \theta(t|\tau) < 1$, but that are not completely monotone. Furthermore, none of these moment sequences are observationally equivalent to a completely monotone moment sequence. This justifies testing for complete monotonicity.

²¹A necessary and sufficient condition is that the moment sequence is nonnegative, *i.e.* that the implied expectation of any nonnegative polynomial in \mathbf{v} is nonnegative. Any completely monotone sequence is nonnegative, so the nonnegativity conditions are a subset of the conditions for complete monotonicity that do not depend on \bar{v} (Widder, 1946).

²²See Shohat and Tamarkin (1943); *e.g.* $\gamma_3 < \gamma_2^2$ would imply $\Pr(v < 0) > 0$. Similar conditions can be derived for γ_i with $i > 5$.

²³Note that the Cauchy-Schwarz inequality implies that for nonnegative variables $\mu_3^2 \leq \mu_2\mu_4$, so if the model is correct then the first inequality is implied by the second.

v in the following way: $\theta(t|\tau, \mathbf{v}) = \psi_1(t, \mathbf{v})\psi_2(\tau)$ with $\psi_1(0, \mathbf{v}) = \mathbf{v}$ and $\psi_1(1, \mathbf{v}) = 1/\mathbf{v}$.²⁴ It can be shown that then the estimate of γ_2 asymptotically is smaller than one. Also, the tests may detect misspecification of the unit of time period. If in reality the model is correct for monthly periods but it is assumed to be correct for quarterly periods, then this may turn up in the γ estimates being inconsistent as moments of a distribution with bounded nonnegative support.

3 Data

We use quarterly unemployment data over the period 1982.IV–1994.IV, collected by the French public employment offices (A.N.P.E.), and subsequently collected on a nation-wide scale by the Department of Labor of the Ministry of Social Affairs and Employment. The data provide the number of unemployed at the *last* day of each quarter who have completed a given number of quarters of unemployment duration in their current spell. So, for example, the data for 1982.IV ($\tilde{U}(t|\tau_0)$) provide the number of individuals who have been unemployed for more than t and less than $t + 1$ quarters at December 31, 1982, and the data for 1983.I ($\tilde{U}(t|\tau_0 + 1)$) similar information for March 31, 1983, etcetera.²⁵ Unemployment here refers to individuals without employment who are immediately available for employment and actively searching for full-time permanent jobs.²⁶

The development of unemployment in France over the data period, adjusted for seasonal effects, is shown in Figure 1. The overall pattern in the development of unemployment of males and females is similar, but there are also some differences. In the first half of the 1980s there is an increase in both unemployment numbers. In the second half of the 1980s male unemployment declines, while female unemployment remains at the same level. In the early 1990s male unemployment catches up with female unemployment, and both male and female unemployment rise. Over the period considered the total relative increase in unemployment is about 60% for both gender groups.

We further clarify the cyclical nature of unemployment by a comparison to two conventional business cycle indicators, the capacity utilization ratio (CUR) and the vacancies-unemployment ratio (VUR). Figure 2 graphs $\ln(\text{VUR})$ and $\ln(\text{CUR}/(1 - \text{CUR}))$, both detrended, deseasonalized and in deviation from their respective means.²⁷ Both indica-

²⁴Note that this is observationally equivalent to a model in which the individual \mathbf{v} changes as a function of duration.

²⁵In our empirical model, we let τ_0 coincide with the first season of a model year. Consistent with this setup, we label the years by 1982.IV–1983.III, 1983.IV–1984.III, \dots , instead of the more generic $0, 1, \dots$ used before. Also, we denote the seasons by IV, I, II, III instead of the more generic $1, \dots, S$.

²⁶‘Immediately’ here means within 15 days, and ‘full-time’ refers to more than 30 hours per week.

²⁷Source of the raw series: Main Economic Indicators of the OECD. Note that the vacancies and unemployment series are measured at the last day of each quarter, whereas the capacity utilization series are measured at the last day of the first month of each quarter. For convenience, we have plotted both series at the last day of each quarter. Note the difference between the horizontal axes of this and the

tors fall until the end of 1981. After this, the CU ratio is almost flat for a few years, whereas the VU ratio increases until early 1983, and decreases sharply until the end of 1984. Both series agree on the business cycle from the mid-1980s onwards: aggregate conditions first improve until the summer of 1990, then deteriorate until the end of 1993, and finally improve again until the end of the data period.

As the cyclical fluctuations in unemployment are somewhat obscured by the large trends we again plot aggregate unemployment, but now both deseasonalized and detrended, in Figure 3. Female and male unemployment are both countercyclical in qualitatively similar ways, but fluctuations are particularly strong for males. Clear peaks in unemployment can be identified at the end of 1984, early 1987, and late 1993/early 1994. Major troughs in unemployment are found early 1983, late 1985/early 1986, and late 1990. If anything, female unemployment lags one quarter after male unemployment. Most of these peaks and troughs are picked up by troughs and peaks in the business cycle indicators. However, the CU ratio hardly peaks in 1983, and neither indicator drops much near 1987, even though the VU ratio is closely related to unemployment itself.

As explained in Subsection 2.1, our model and estimation method are designed to be applicable to quarterly exit probabilities computed from the discrete time unemployment duration data. Consistent with this design we use the number of unemployed in the first duration class ($U(0|\tau)$) as the measure of the size of the inflow into unemployment. This variable is smaller than the continuous time inflow because it excludes the persons who enter and leave unemployment between two measurement points.²⁸ The corresponding discrete time measure of the outflow from unemployment at time τ is the difference between the inflow into unemployment at time $\tau + 1$ and the growth of the stock of unemployed between τ and $\tau + 1$, or $U(0|\tau + 1) + U(\tau) - U(\tau + 1)$. For later reference, note that the discrete incidence measure $U(0|\tau)$ can actually be thought to correspond to inflow between $\tau - 1$ and τ , whereas the outflow at τ , and the exit probabilities $\theta(t|\tau)$, refer to exits between τ and $\tau + 1$ instead. This has implications for the interpretation of seasonal patterns. For instance, the seasonal effects on the size ($\omega_4(\text{IV})$) and composition ($\omega_3(\text{IV})$) of the inflow at December 31 correspond to the inflow during the fourth quarter, whereas the contemporaneous seasonal effect on the exit probability ($\omega_2(\text{IV})$) affects the outflow during the first quarter of the next year.

Figure 4 shows the development of the inflow into unemployment after adjusting for seasonal effects. Again, the general pattern in the fluctuations is the same for males and

following graphs.

²⁸In the literature both measurement methods have been used. For example, Sider (1985) uses the latter whereas Layard, Nickell and Jackman (1991) use the same method as we do. According to Jackman and Layard (1991) the different measures exhibit similar behavior over the business cycle. In our case, from additional analysis it appears that many of the dynamic features of both series are similar, with seasonal fluctuations dominating cyclical and secular developments. It does seem that in the late 1980s the true inflow has increased more than the number of unemployed in the first duration class. This may however be caused by a change in the data collection procedure. We return to this issue below.

females. However, the trends are clearly different. In the beginning of the 1980s, the female inflow starts well below the male inflow, but, although the inflow of males into unemployment is slightly increasing over time, the inflow of females shows a much stronger increase. In the beginning of the 1990s, male and female inflow figures are roughly similar. Then, the male inflow again rises above the female inflow. Figure 5 shows the development of the outflow from unemployment. The general pattern in the outflow is very similar to the one in the inflow. Until 1988 the outflow from female unemployment is smaller than from male unemployment. After that the outflow figures are about the same and are both somewhat decreasing. In the early 1990s, the male outflow rises again relative to the female outflow. Obviously, the increase in unemployment is due to a positive difference in the sizes of the inflow and the outflow. Over the period the average quarterly outflow from male unemployment is 30.0% of total male unemployment, the average inflow is 30.7%. For females these figures are 26.5% for the outflow and 27.0% for the inflow.

Now let us consider the relation in the data between exit probabilities out of unemployment and unemployment duration. Figure 6 shows this relationship at different points of (calendar) time. At every point of time the exit probability declines over the duration of unemployment. This decline can be due to unobserved heterogeneity, individual negative duration dependence, or a combination of both. For females, the exit probability ranges from 0.31 to 0.37 in the first quarter to 0.10 to 0.17 after 4.5 years of unemployment. The exit probability for males is somewhat higher in the first quarters of unemployment, but approximately the same for higher duration classes. The exit probabilities for quarter 1992.I are lower than those in the quarters 1983.I and 1987.I and are also less steeply decreasing. So, the higher the exit probability in the beginning, the steeper the decline in the exit probability over the duration of unemployment. This suggests that unobserved heterogeneity is an important factor in French unemployment dynamics. If the exit probability from the first duration class is high, then the dynamic sorting within a cohort of unemployed is faster, causing the average exit probability to decline more rapidly over the duration of unemployment.²⁹

In 1986, some details of the procedure according to which the data are collected were changed. As a result, the time series on $\tilde{U}(t|\tau)$ exhibit ruptures at 1986.IV. This turns out to be particularly important for the series on $\tilde{U}(0|\tau)$. Further, the French policy towards youth unemployment changed substantially in the mid-1980s as well. The new policy basically entailed that young individuals were assigned to training jobs shortly after entering unemployment. This may be expected to affect the exit probability out of the first duration class $\theta(0|\tau)$. For these reasons, we add to the model a dummy variable

²⁹Abbring, Van den Berg and Van Ours (1994, 1997) use this result on the interaction of duration dependence and the business cycle at the aggregate level to distinguish between dynamic sorting caused by heterogeneity, *i.e.* the MPH model we have here, and the ranking model of Blanchard and Diamond (1994), which produces other types of interaction. Also recall that the interaction is crucial for the identification of heterogeneity and individual duration dependence in our model (see Subsection 2.3).

$d(\tau)$ equalling one if τ is after 1986.IV and zero otherwise. Specifically, we multiply the expressions for $U(0|\tau)$ and $\theta(0|\tau)$ in the corresponding model equations by $\exp(d_{\geq'87} \cdot d(\tau))$, in which $d_{\geq'87}$ is a parameter to be estimated. Although this notation suggests otherwise, we do not impose $d_{\geq'87}$ to be the same in the equation for $U(0|\tau)$ and the equation for $\theta(0|\tau)$. The results turn out to be insensitive with respect to small changes of the calendar time point defining the areas in which the dummy variable equals zero and one, respectively.

4 Estimation results

4.1 Some preliminary issues

We estimate our model using observations of unemployment in the first six duration classes, *i.e.* series of observations $\tilde{U}(0|\tau), \dots, \tilde{U}(5|\tau)$. This allows us to compute five quarterly exit probabilities from unemployment (*i.e.*, $K = 4$), and to estimate a five equation duration model, as given by equation (7), for each gender type separately. It turns out to be difficult to estimate models that include equations for exit probabilities from higher duration classes. Note that, as t increases, the degree of complexity and nonlinearity of $\theta(t|\tau)$ as a function of the parameters increases enormously. Below we show, however, that the estimation results can be used to make certain inferences on unemployment dynamics in higher duration classes. Estimation of incidence equation (8), using incidence observations $\tilde{U}(0|\tau)$, completes the estimation.

In order to detect the correlation structure of the measurement errors $\ln \varepsilon_{t,\tau}$ over t and τ we analyze the estimates of the errors $e_{t,\tau}$ of the equations (3), from an estimation with supposedly uncorrelated measurement errors. To make inferences, the correlation structures of the second type of errors has to be expressed in terms of those of the first type. If the $\varepsilon_{t,\tau}$ are i.i.d. then the only nonzero correlation in the $e_{t,\tau}$ concerns $\text{corr}(e_{t,\tau}, e_{t+1,\tau+1})$, which equals $-\frac{1}{2}$ for every t and τ . As argued in Section 2, there are various reasons for $\ln \varepsilon_{t,\tau}$ and $\ln \varepsilon_{t+1,\tau}$ to be correlated for every t ; some of these reasons implying a positive correlation and other a negative. In such cases the largest correlation between the errors of the equations (3), apart from the one noted above, is $\text{corr}(e_{t,\tau}, e_{t+1,\tau})$, which has the same value as $\text{corr}(\ln \varepsilon_{t,\tau}, \ln \varepsilon_{t+1,\tau})$. Similar results can be derived for other a priori plausible correlation schemes.

The residual analysis seems to suggest that there are nonzero (positive) correlations between measurement errors $\ln \varepsilon_{t,\tau}$ at one single calendar moment. We find no evidence for other correlation schemes like serial correlation over calendar time. Thus, we specify $\text{corr}(\ln \varepsilon_{t^*,\tau^*}, \ln \varepsilon_{t^{**},\tau^{**}}) = r^{|t^*-t^{**}|}$ if $\tau^* = \tau^{**}$, and 0 otherwise, with $-1 < r < 1$.

Table 1 gives estimates of the duration model with correlated measurement errors for both males and females. Residual analysis based on these estimates supports the specification of the measurement errors. The estimates of the correlation parameter indicate that

the errors are positively correlated across duration classes at one calendar moment. We may conclude that misclassification of unemployed individuals into wrong duration classes is not a major source of errors in the observed unemployment figures. Indeed, standard deviations of the measurement errors slightly below 0.02 show that measurement errors in unemployment numbers are generally small. Durbin-Watson statistics are satisfactory. The pseudo- R^2 statistics reveal a very good fit, especially for the lower duration classes and for males.

In the remainder of this section, we first discuss the estimates of the duration dependence and heterogeneity parameters, and the results of the model specification tests. Then, we discuss the trends and cyclical effects, and the seasonal effects, in both the incidence and the exit probabilities. We conclude with some simulations to illustrate the role of the various components in unemployment dynamics.

4.2 Unobserved heterogeneity and duration dependence

The estimates of the individual duration dependence parameters indicate that there is significant non-monotonous duration dependence for both gender types. The effect of individual duration dependence on the exit probability from the first five duration classes is shown in Figure 7. Individual female unemployed face a significant 17% rise in their exit probability after one quarter of unemployment, a small return to about 104% of the initial level in the next quarter, and further increases in the next two quarters, *ceteris paribus*. Male individual duration dependence follows a qualitatively similar pattern, except for some negative duration dependence between quarters 2 and 3. Overall, male duration dependence is slightly more negative. Clearly, these results are not consistent with stigma, loss of skills, or demotivation effects on exit probabilities in the first five quarters of unemployment. Furthermore, these non-parametric estimates are not compatible with frequently used monotonous parameterizations of genuine duration dependence like the Weibull function. Lollivier (1994) finds that there is genuine negative duration dependence from duration zero onwards, and that it is particularly strong when going from the second to the third quarter. This is in accordance to our results for duration classes 1 and 2.

The results on genuine duration dependence imply that the decrease of the observed exit probabilities during the first five quarters of unemployment are mainly due to unobserved heterogeneity. Indeed, the estimates in the first five rows of Table 1 imply significant heterogeneity. For both males and females the second normalized moment γ_2 is significantly larger than 1, which implies a positive variance of the heterogeneity distribution $G(v)$. This is consistent with the higher normalized moments being significantly larger than one.

As the resulting dynamic selection is stronger in booms, the aggregate exit probabilities decrease faster with duration in booms than in recessions. Figure 8 illustrates this effect of the unobserved heterogeneity on average exit probabilities at two different states of

the business cycle, abstracting from all other effects, *i.e.* from duration dependence and seasonal effects, and assuming that the business cycle effect is the same for the five exit probabilities. We choose the absolute deviation of the high and the low values of $\ln \theta$ from the mean value to correspond to one standard deviation of its yearly components at $t = 0$.³⁰ For expositional purposes we normalize the corresponding graphs to 0 at $t = 0$. For both females and males, the decrease of the exit probabilities is stronger at the top of the cycle. It turns out that the selection due to heterogeneity is most severe in the early stages of unemployment, although there seems to be a further large selection effect in the last duration classes. The latter effect may not be significant, given the large standard errors of the estimates of the higher moments. Indeed, the moment-inequality statistics show that we can restrict the moments to belong to discrete distributions with two points of support, in which case the latter effect disappears.³¹

We can also exploit this last result to informally extend our analysis to higher duration classes, without estimating a fully non-parametric model on data covering these duration classes. In Figure 6 we have seen that aggregate exit probabilities continue to decrease after the first 5 duration classes, to which we have restricted our analysis so far. To provide a first impression of the role of unobserved heterogeneity and genuine duration dependence in explaining this negative duration dependence beyond the first 1.5 years of unemployment, we extrapolate our model in the following way. First, we estimate the parameters of the discrete heterogeneity distributions. Fixing all but the heterogeneity parameters at their non-parametric estimates, and maximizing the restricted likelihood with respect to the heterogeneity parameters only, we find shares of 82.5% at 0.217 and 17.5% at 0.541 for females, and masses of 87.7% at 0.274 and 13.3% at 0.679 for males.³² Next, if we assume that the higher moments, which we have not considered so far, also correspond to the same discrete distributions, we can extrapolate the effects of dynamic sorting beyond the fifth duration class. We find that most of the dynamic selection due to the two point heterogeneity is completed at the sixth quarter. As there is substantial observed negative duration dependence after the sixth quarter, we conclude that individual duration dependence turns negative after 1.5 years of unemployment.

Finally, we turn to the moment-inequality specification tests we proposed in Subsection 2.5. It turns out that neither of the tests based on equations (12) results in a rejection, for males and females alike. This is an important result since, as we have seen, these

³⁰The relevant standard deviations of $\alpha_2(y(\tau)) + \alpha_3(y(\tau))$ are 0.091 and 0.106 for females and males, respectively.

³¹Shohat and Tamarkin (1943) and Lindsay (1989) discuss the relation between the moment problem, and therefore the moment-inequality statistics, and discrete distributions. See also Abbring, Van den Berg and Van Ours (1994).

³²Similar results are found if we minimize the squared distance of the moments of a two point discrete distribution to the non-parametric moment estimates, weighted by the ML covariance matrix. The resulting minimum distances are not significant, which confirms the results from the moment-inequality tests.

moment-inequality tests have high power against a wide range of model alternatives. As was noted in Subsection 2.5, the model also implies bounds for the exit probability out of the first duration class in terms of γ . The standard errors of these bounds turn out to be rather high, and observed exit probabilities out of the first duration class are well within the confidence intervals of these upper bounds. This again supports our MPH model.

4.3 Trends and cycles

In this subsection we discuss the estimated trends in the components of unemployment, and assess the cyclical nature of each of the components by comparing the estimated cycles to the business cycle indicators of Section 3.

Table 1 shows that exit probabilities vary significantly over calendar time. First, we find that both female and male exit probabilities trend downward significantly, at rates equal to -2.0% and -2.4% yearly, respectively. Second, according to the Wald statistics, there is significant contemporaneous cyclical variation and significant cyclical variation in the composition of the inflow (*i.e.*, between cohorts), where the first dominates statistically. In Figure 9 the contemporaneous cycles in the exit probabilities are plotted. Both cycles are roughly procyclical, decreasing in the early 1980s, increasing in the late 1980s, and again decreasing in the early 1990s. However, the initial fall in female exit probabilities is sharper and longer, and female probabilities seem to recover earlier in the 1990s. Regressions on the indicators confirm this rather mixed picture of procyclical exit probabilities. The male contemporaneous cycle is well explained by the 5 quarter leading CU ratio, giving an $R_a^2 = 0.71$ and an estimate on the CU ratio of 0.34 (0.06), whereas the VU ratio performs badly. The female cycle, on the other hand, is picked up quite well by the same period VU ratio, giving $R_a^2 = 0.39$ and a VU ratio effect of 0.14 (0.05). The regressor effects are robust among the regressions with leading or lagged indicators that have some explanatory power. In general, this holds for the regressions discussed in the sequel.

Figure 10 plots the cohort cycles, which are thought to represent cyclical variation in the unobserved composition of the inflow into unemployment. In particular for males, the slightly procyclical cohort effects are small compared to the contemporaneous and incidence cycles. This is confirmed by regressions, robustly giving a 0.04 (0.01) estimate on the 2 quarters lagged VU ratio for males ($R_a^2 = 0.40$), and a 0.23 (0.06) on the 2 quarters leading CU ratio for females ($R_a^2 = 0.58$).

Table 2 presents the estimates of the equation for the size of the inflow into unemployment. First, note that the incidence equation fits the data very well. We find significant positive trends in both female ($2.7\%/year$) and male ($1.8\%/year$) incidence. Note that the female trend in the incidence is larger, whereas the male trend in the outflow dominates.³³ A possible explanation for the slightly stronger trend in female incidence is the

³³See Cohen and Lefranc (1994) for an analysis of trends in French unemployment.

development of labor market participation: whereas the female participation rate grew by a few percentage points, the male rate fell somewhat in the data period considered.

Cyclical fluctuations are significant as well, in particular of male incidence. Figure 11 shows the estimated cyclical variation of the incidence. In particular for females, cyclical variation is fairly limited compared to the seasonal fluctuations and the trends. The female and male cycle are similar until 1987/1988, and agree on a (countercyclical) decreased incidence from 1993 onwards. However, only male incidence is clearly countercyclical in the period between 1988 and 1993; the cycle in the female incidence is fairly flat. Regressions on the business cycle indicators reveal a clear overall countercyclical pattern in male incidence: a CU ratio leading two quarters gives an adjusted R -squared (R_a^2) of 0.34, and a parameter on the CU ratio series of -0.20 (s.e. 0.07).³⁴ Regressions of the female cycle on the CU ratio are less significant and less clear about the appropriate lag or lead, but, if anything, female incidence seems to be mildly procyclical: a variety of lags and leads gives business cycle effects of around 0.14 (0.07), with $R_a^2 \approx 0.1$ –0.3. This surprising result can possibly be traced back to a relatively high share of labor market entrants, as opposed to job losers, in female unemployment incidence. In turn, this would be consistent with the relative increase in female labor market participation. Note that the cycles in the incidence cannot be explained very well by the VU ratio. Comparing Figures 9 and 11 and the regression results, we find that contemporaneous cyclical fluctuations in the exit probabilities are somewhat larger than cyclical fluctuations in the incidence.

In sum, the business cycle influences unemployment foremost by procyclical contemporaneous effects on individual outflow probabilities. For males, we also find a significant contribution of the incidence to countercyclical unemployment fluctuations, but only a very small procyclical cohort effect on the exit probabilities. For females, the cyclicity of the incidence is less determined, and even seems to be dampening the cyclicity of unemployment. We find a stronger cohort effect than for males, but, even though it is tested to be procyclical, it mainly follows the overall increase in the indicators until the start of the 1990s, and the overall decrease thereafter, and does not explain much of the more subtle fluctuations in female unemployment.

It may be interesting to compare these results to other studies, even though most of those are based on U.S. or U.K. data and some of them use data that cover only a small time span and/or only a specific subset of the population. There have been numerous studies examining the relative importance of incidence and duration to explain variation in unemployment. The evidence is mixed, and results differ between different countries and time periods. Layard, Nickell and Jackman (1991) present a survey based on aggregate

³⁴We create yearly indicator series with various quarterly lags by selecting data on one particular season in each of the years only, and throwing away data on the remaining 3 seasons. Then, the year dummies are regressed on a yearly indicator series, a constant, and a linear trend. The indicator series are log (VU ratio) or inverse logit (CU ratio) transformed, but not detrended, which is the reason for including a linear trend.

data, from which it can be concluded that for most European countries, including France, the variation in unemployment duration is more important than the variation in incidence in explaining total variation in unemployment over the business cycle.³⁵

There has also been some debate on whether the business cycle effect on durations works by way of an effect on the composition of the inflow or by way of a direct effect on the outflow probabilities. Darby, Haltiwanger and Plant (1985) argue that two groups of individuals can be distinguished, one group with high transition rates into and out of unemployment, and one with a high degree of specific human capital and with long-duration jobs and high unemployment durations. In a recession, firms in declining industries find it optimal to accelerate labor force reductions, and the inflow into unemployment will consist to a relatively large extent of individuals in the second group. In terms of our model this means that α_3^\perp should vary procyclically. However, we find that male cohort effects are very small, and that female cohort effects, although somewhat larger, only roughly track the business cycle. This is consistent with the empirical work of Dynarski and Sheffrin (1990), Imbens and Lynch (1992) and Baker (1992a). The first two studies use micro data while the third uses aggregate data containing a large number of observed explanatory variables. In these cases, certain attributes of the inflow are directly observed. All conclude that the business cycle affects individual outflow probabilities, and, in the case of the two last-mentioned papers, that the composition of the inflow is more or less constant, so that the business cycle effect on unemployment duration works primarily by affecting the individual outflow probabilities contemporaneously.

In the empirical literature there is a large agreement on the sign of the relation between observed outflow probabilities and the state of the business cycle. Butler and McDonald (1986), Dynarski and Sheffrin (1990), Imbens and Lynch (1992), Baker (1992a) and Lollivier (1994) all conclude that this sign is positive, and that therefore individual outflow probabilities are procyclical. This result is justified theoretically in Van den Berg (1994), who generalizes previous theoretical papers by showing that in job search models the job offer arrival rate has a positive effect on the exit rate out of unemployment, under almost every possible configuration of model determinants.

4.4 Seasonal effects

Finally, Tables 1 and 2 provide estimates of the seasonal effects in the components of unemployment. There are strong and highly significant seasonal fluctuations in the incidence. For females (males) the incidence in the top season is 50% (39%) higher than the incidence in the bottom season. Incidence is relatively large in the second half of the year. One obvious reason for this is the fact that individuals usually leave school and enter the labor market in the third quarter.

³⁵See also Sider (1985) and Pissarides (1986).

We also find less pronounced but significant seasonal variation of the exit probabilities. Exit probabilities are 9–10% higher in the top contemporaneous and cohort seasons for females, and 9% higher in the top seasonal cohort and 15% higher in the top contemporaneous season for males. Individual exit probabilities in any cohort are highest at (the last day of) quarter I and lowest at (the last day of) quarter IV. Recalling the discussing in Section 3, this implies that relatively many exits occur during the second quarter and relatively few during the first quarter of each year. The composition of the inflow is relatively favorable in the last two quarters of the year. This is not surprising since the inflow in those quarters consists to a large extent of individuals leaving school.

Note that the seasonal effects are rather similar between males and females. The main difference is that female incidence is relatively high in the third quarter. This may be due to layoffs of females from summertime service sector jobs. For females, seasonal fluctuations in the incidence are relatively large compared to the fluctuations in the exit probabilities.

There is a positive relation between the size and the quality of a cohort over seasons: if the inflow is large, the average exit probability of the inflowing cohort is high and *vice versa*. This is not an obvious relationship, as one might expect more competition for jobs between the individuals in larger cohorts, and therefore smaller individual exit probabilities for larger cohorts. However, if individuals also compete for jobs with individuals from other cohorts then one should expect the overall exit probabilities to decline in or just after large cohort seasons. The results provide some evidence for the latter: for instance, male incidence is much lower during the first half the year, and the overall exit probability is relatively high between the end of the first and the second quarter ($\omega_2(I)$ is relatively high for males). However, high exit probabilities during the second quarter (*i.e.*, high $\omega_2(I)$) could also be due to inflow into summer employment. Finally, the seasonal inflow in the third quarter is largely due to school leavers, which are more than averagely equipped to find a job, and thus have a higher than average exit probability.

Visser (1992) and Lollivier (1994) estimate models for the exit rate out of unemployment using French micro data, taking account of seasonal effects on the exit rate. Visser (1992) finds that the exit rate is largest in the first and the third quarter, which confirms our results. Lollivier (1994), who uses monthly data, finds that the exit rate is relatively large in the fourth quarter and in January, and very small in July. It should be noted that in both studies the data cover only a limited time span.

4.5 Some simulations

Figures 9–11 show the main cyclical components of unemployment, but it is difficult to assess the relative contribution of each determinant to unemployment dynamics. The same holds for the various seasonal effects. Since we study the exit probabilities from unemployment in the first five duration classes, it is not possible to draw conclusions

about unemployment dynamics for all possible durations.³⁶ In Figure 12 we present the results of some simulations to illustrate the differences in the effects of the quality of inflow, the incidence and the contemporaneous time effects. We take the average quarterly inflow into unemployment, *i.e.* the number of unemployed in the first duration class, equal to 100. Then we calculate for different situations how many workers are still unemployed after one year, and end up in $U(4|\tau)$. We use the estimated parameters for the unobserved heterogeneity distribution and the individual duration dependence to calculate the evolution of the average exit probability over the duration of unemployment.³⁷

First, we determine the effect of different steady state levels of the exit probabilities. For females we distinguish between steady states in which the mean outflow probabilities at $t = 0$ equal 0.256, 0.281 and 0.308.³⁸ Then, the number of unemployed in the fifth duration class ranges from 25.0–31.9. For males we use steady state exit probabilities at $t = 0$ of 0.284, 0.316 and 0.351. Then, the number of unemployed in the fifth duration class varies between 21.6 and 29.7.

Next, for the medium steady state exit probabilities, we calculate the effect of seasonal fluctuations. We first shortly discuss the effect of different sequences of contemporaneous seasonal effects. As individuals experience a full seasonal cycle between entering unemployment and ending up in the fifth duration class, any effect of the contemporaneous seasonal effects stems from the interaction with individual duration dependence, and is very small. Seasonal variation in the quality of the inflow into unemployment does imply seasonal variation in the number of unemployed in the fifth duration class. For females, we find outcomes between 26.6 and 30.1, for males this range is 24.0–27.3. Seasonal fluctuations in the incidence imply the largest seasonal fluctuations in the number of unemployed in the fifth duration class, between 24.3 and 36.4 for females and 21.1–29.4 for males.

5 Conclusion

In this paper we shed a light on unemployment dynamics by decomposing aggregate unemployment data. We develop and estimate a flexible model which allows the size of the inflow, the composition of the inflow, and the individual exit probabilities to depend on the state of the business cycle as well as the prevailing season. Moreover, we allow for unobserved heterogeneity and measurement errors in the data. We prove non-parametric identification of the model and develop and apply specification tests. We apply the method to quarterly French unemployment data on the period 1982.IV–1994.IV. The model spec-

³⁶The exercise in the previous subsection was rather informal. Moreover, there are no data on the 20th and higher duration classes.

³⁷Note that we again use the unrestricted ML estimates.

³⁸The corresponding log exit probabilities are one standard deviation below the mean, the mean, and one standard deviation above the mean of $\ln\theta(0|\tau)$ over τ . Here, we again use standard deviations of the long run part of $\ln\theta(0|\tau)$ ($\alpha_2(y(\tau))\alpha_3(y(\tau))$).

ification is accepted and the results are robust with respect to various assumptions.

The relevance of the empirical results is threefold. First of all, the results on business cycle effects have implications for the plausibility of existing as well as future theoretical macroeconomic models of unemployment. Second, the magnitude of the business cycle and seasonal effects on the exit probabilities is such that they should be taken into account in standard micro-econometric unemployment duration analyses. Third, the results on the way in which business cycles affect unemployment, and the results on the role of unobserved heterogeneity versus genuine duration dependence, are of interest for unemployment policy.

We now summarize the main empirical conclusions. First, the state of the business cycle influences unemployment mainly by way of affecting the individual outflow probabilities, and to a lesser extent the size of the inflow. Cohort effects, thought to work via the composition of the inflow, are less important. Individual outflow probabilities are procyclical, the exit probability in the top of the cycle being, at a maximum, about 22–28% larger than the exit probability in a recession, *ceteris paribus*. The composition of the inflow is procyclical, but male cohort effects are very small, and female cohort effects only roughly track business cycle fluctuations. Male incidence is countercyclical, but female incidence, if anything, procyclical. There is also a downward trend in the exit probabilities, reflecting part of the rise in French unemployment over the data period. The remaining trend in French unemployment is due to an upward trend in the incidence, which is particularly strong for females.

In general, there are very large seasonal effects on the size of the inflow. The differences across seasons can be as large as 50% of the smallest effect. There are also smaller seasonal effects on the average quality of the inflow into unemployment and on the individual exit probabilities. Again, variation in the incidence is relatively important for females. Also, the size of the inflow in any season is positively correlated with the average quality of the inflow.

There is no negative duration dependence of individual exit probabilities during the first 1.5 years of unemployment. Thus, stigma effects seem to be absent at those durations, and the decrease of the observed exit probabilities is almost completely due to the dynamic selection induced by unobserved heterogeneity. It turns out that this dynamic selection is mostly completed by the end of the 1.5 year period, suggesting that observed (aggregate) duration dependence at higher durations is explained by negative individual duration dependence. Finally, because of the unobserved heterogeneity, the observed exit probabilities decrease less fast in a recession than in the top of the cycle. The latter is a robust feature of the data. This contradicts alternative models that predict an opposite interaction, like the ranking model.

Appendix: Proof of Proposition 1

We first introduce some notation. Let μ be the moment operator with respect to the moment sequence $\{\mu_i\}$, i.e. $\mu(\sum_i \beta_i v^i) = \sum_i \beta_i \mu_i$ for any sequence of constants $\{\beta_i\}$ (Widder, 1946). Similarly, let $\hat{\mu}$ be the moment operator with respect to the sequence $\{\hat{\mu}_i\}$. Define

$$\nu(\tau, t) = \frac{\mu \left(v \prod_{i=0}^{t-1} (1 - \psi_1(i) \psi_2(\tau - t + i) \psi_3(\tau - t) v) \right)}{\mu \left(\prod_{i=0}^{t-1} (1 - \psi_1(i) \psi_2(\tau - t + i) \psi_3(\tau - t) v) \right)},$$

and similarly define $\hat{\nu}$ in terms of $\hat{\mu}$, $\hat{\psi}_1$, $\hat{\psi}_2$, and $\hat{\psi}_3$. Finally, let $H(\tau, t) = \prod_{i=0}^t (1 - \theta(\tau - t + i, i))$ and $\hat{H}(\tau, t) = \prod_{i=0}^t (1 - \hat{\theta}(\tau - t + i, i))$.

Observational equivalence of ψ and $\hat{\psi}$ implies that

$$\forall(\tau, t) \in \mathcal{T}_{N,K} : \psi_1(t) \psi_2(\tau) \psi_3(\tau - t) \nu(\tau, t) = \hat{\psi}_1(t) \hat{\psi}_2(\tau) \hat{\psi}_3(\tau - t) \hat{\nu}(\tau, t), \quad (13)$$

and, equivalently, that $\forall(\tau, t) \in \mathcal{T}_{N,K} : H(\tau, t) = \hat{H}(\tau, t)$. Note that Assumptions 1–3 imply that $0 < \theta(\tau, t) < 1$ and, by implication, $0 < H(\tau, t) < 1$ for all $(\tau, t) \in \mathcal{T}_{N,K}$. Our proof proceeds in 3 steps.

- (i). Because of Assumption 6 it is possible to find a $\tau_1, \tau_1 + S \in T_N(1) : \theta(\tau_1 - 1, 0) \neq \hat{\theta}(\tau_1 - 1 + S, 0)$. Let $\tau^* = \tau_1$ if $s(\tau_1 - 1) = 1$, and $\tau^* = \tau_1 - 1$ otherwise. Because of Assumption 4, evaluating (13) at τ^* and $\tau^* + S$ and dividing gives

$$\frac{\alpha_2(\tau^*) \alpha_3(\tau^* - t) \nu(\tau^*, t)}{\alpha_2(\tau^* + S) \alpha_3(\tau^* + S - t) \nu(\tau^* + S, t)} = \frac{\hat{\alpha}_2(\tau^*) \hat{\alpha}_3(\tau^* - t) \hat{\nu}(\tau^*, t)}{\hat{\alpha}_2(\tau^* + S) \hat{\alpha}_3(\tau^* + S - t) \hat{\nu}(\tau^* + S, t)},$$

which, evaluated at $t = 0$ and $t = 1$, implies that

$$\frac{\nu(\tau^*, 1)}{\nu(\tau^* + S, 1)} = \frac{\hat{\nu}(\tau^*, 1)}{\hat{\nu}(\tau^* + S, 1)},$$

as $y(\tau^*) = y(\tau^* - 1)$ and $y(\tau^* + S) = y(\tau^* + S - 1)$. Multiplying by $H(\tau^*, 0)/H(\tau^* + S, 0) = \hat{H}(\tau^*, 0)/\hat{H}(\tau^* + S, 0)$, and substituting $\theta(\tau, 0)$ and $\hat{\theta}(\tau, 0)$ gives

$$\frac{1 - \theta(\tau^*, 0) \gamma_2}{1 - \theta(\tau^* + S, 0) \gamma_2} = \frac{1 - \hat{\theta}(\tau^*, 0) \hat{\gamma}_2}{1 - \hat{\theta}(\tau^* + S, 0) \hat{\gamma}_2}. \quad (14)$$

The left and right hand sides of (14) are the same function of γ_2 and $\hat{\gamma}_2$, respectively. Under Assumption 6 this function is strictly monotonous, implying that $\gamma_2 = \hat{\gamma}_2$.

- (ii). In turn, as we can write

$$\frac{\nu(\tau, t)}{\mu_1} = \frac{\mu \left(\frac{v}{\mu_1} \prod_{i=0}^{t-1} \left(1 - \frac{\theta(\tau - t + i, i)}{\nu(\tau - t + i, i)} \mu_1 \frac{v}{\mu_1} \right) \right)}{\mu \left(\prod_{i=0}^{t-1} \left(1 - \frac{\theta(\tau - t + i, i)}{\nu(\tau - t + i, i)} \mu_1 \frac{v}{\mu_1} \right) \right)}, \quad (15)$$

and similarly express $\hat{\nu}(\tau, t)/\hat{\mu}_1$ in terms of $\hat{\mu}_1$, $\hat{\nu}$ and $\hat{\mu}$, this implies that $\forall \tau \in T_N : \nu(\tau, 1)/\mu_1 = \hat{\nu}(\tau, 1)/\hat{\mu}_1$. Thus, for $t = 0$ and $t = 1$, (13) can be reduced to

$$\forall \tau \in T_N(t) : \psi_1(t) \psi_2(\tau) \psi_3(\tau - t) \mu_1 = \hat{\psi}_1(t) \hat{\psi}_2(\tau) \hat{\psi}_3(\tau - t) \hat{\mu}_1. \quad (16)$$

Evaluating (16) at any $\tau, \tau' \in T_N(t)$ and dividing gives

$$\forall \tau, \tau' \in T_N(0) : \frac{\psi_2(\tau) \psi_3(\tau)}{\psi_2(\tau') \psi_3(\tau')} = \frac{\hat{\psi}_2(\tau) \hat{\psi}_3(\tau)}{\hat{\psi}_2(\tau') \hat{\psi}_3(\tau')} \text{ for } t = 0, \text{ and} \quad (17)$$

$$\forall \tau, \tau' \in T_N(1) : \frac{\psi_2(\tau)\psi_3(\tau-1)}{\psi_2(\tau')\psi_3(\tau'-1)} = \frac{\hat{\psi}_2(\tau)\hat{\psi}_3(\tau-1)}{\hat{\psi}_2(\tau')\hat{\psi}_3(\tau'-1)} \text{ for } t = 1. \quad (18)$$

Substituting (18) in (17) and iterating gives

$$\forall \tau, \tau' \in T_N(t) : \frac{\psi_3(\tau)/\psi_3(\tau-t)}{\psi_3(\tau')/\psi_3(\tau'-t)} = \frac{\hat{\psi}_3(\tau)/\hat{\psi}_3(\tau-t)}{\hat{\psi}_3(\tau')/\hat{\psi}_3(\tau'-t)} \text{ for any } t \in D_K. \quad (19)$$

Evaluating (13) at any $(\tau, t), (\tau', t) \in T_{N,K}$, dividing, and substituting (19) and (17) shows that

$$\forall (\tau, t), (\tau', t) \in T_{N,K} : \frac{\nu(\tau', t)}{\nu(\tau, t)} = \frac{\hat{\nu}(\tau', t)}{\hat{\nu}(\tau, t)}.$$

Multiplying by $H(\tau', t)/H(\tau, t) = \hat{H}(\tau', t)/\hat{H}(\tau, t)$, and substituting $\theta(\tau, t)$ and $\hat{\theta}(\tau, t)$ gives

$$\begin{aligned} & \forall (\tau, t), (\tau', t) \in T_{N,K} : \\ & \frac{\mu \left(\frac{v}{\mu_1} \prod_{i=0}^{t-1} \left(1 - \frac{\theta(\tau' - t + i, i)}{\nu(\tau' - t + i, i)} \mu_1 \frac{v}{\mu_1} \right) \right)}{\mu \left(\frac{v}{\mu_1} \prod_{i=0}^{t-1} \left(1 - \frac{\theta(\tau - t + i, i)}{\nu(\tau - t + i, i)} \mu_1 \frac{v}{\mu_1} \right) \right)} = \frac{\hat{\mu} \left(\frac{v}{\hat{\mu}_1} \prod_{i=0}^{t-1} \left(1 - \frac{\hat{\theta}(\tau' - t + i, i)}{\hat{\nu}(\tau' - t + i, i)} \hat{\mu}_1 \frac{v}{\hat{\mu}_1} \right) \right)}{\hat{\mu} \left(\frac{v}{\hat{\mu}_1} \prod_{i=0}^{t-1} \left(1 - \frac{\hat{\theta}(\tau - t + i, i)}{\hat{\nu}(\tau - t + i, i)} \hat{\mu}_1 \frac{v}{\hat{\mu}_1} \right) \right)}. \end{aligned} \quad (20)$$

We are now ready to prove by induction that $\nu(\tau, t)/\mu_1 = \hat{\nu}(\tau, t)/\hat{\mu}_1$ on $T_{N,K}$ and $\gamma = \hat{\gamma}$:

- (a) Note that $\forall (\tau, t) \in T_{N,K}(1) : \nu(\tau, t)/\mu_1 = \hat{\nu}(\tau, t)/\hat{\mu}_1$ and $\gamma_t = \hat{\gamma}_t$. Initialize $k = 2$.
 - (b) Suppose that $\forall (\tau, t) \in T_{N,K}(k-1) : \nu(\tau, t)/\mu_1 = \hat{\nu}(\tau, t)/\hat{\mu}_1$ and $\gamma_t = \hat{\gamma}_t$ for $t = 1, \dots, k$. Then, the left and right hand sides of (20) evaluated at $t = k$ are the same function of γ_{k+1} and $\hat{\gamma}_{k+1}$, respectively. It is easy to check that, because of Assumption 6, there is a pair $\tau_k, \tau'_k \in T_N(k)$ for which this function is strictly monotonous.³⁹ So, as (20) also holds for $\tau = \tau_k$ and $\tau' = \tau'_k$, it necessarily follows that $\gamma_{k+1} = \hat{\gamma}_{k+1}$. Then, (15) implies that $\forall \tau \in T_N(k) : \nu(\tau, k)/\mu_1 = \hat{\nu}(\tau, k)/\hat{\mu}_1$.
 - (c) If $k < K$, let $k = k + 1$ and repeat (b).
- (iii). To complete the proof, note that, as a result, (16) holds for all $t \in D_K$:

$$\forall (\tau, t) \in T_{N,K}(\tau, t) : \psi_1(t)\psi_2(\tau)\psi_3(\tau-t)\mu_1 = \hat{\psi}_1(t)\hat{\psi}_2(\tau)\hat{\psi}_3(\tau-t)\hat{\mu}_1. \quad (21)$$

Furthermore, we know from (19) that $\psi_3(\tau)/\psi_3(\tau-1)$ and $\hat{\psi}_3(\tau)/\hat{\psi}_3(\tau-1)$ are proportional, which implies that $\hat{\psi}_3(\tau) = \hat{a}f^\tau\psi_3(\tau)$ for some constants $\hat{a}, f > 0$. Also, we know from (17) that $\psi_2(\tau)$ and $\hat{\psi}_2(\tau)f^\tau$ are proportional given t , implying that $\hat{\psi}_2 = \hat{b}f^{-\tau}\psi_2$ for some constant $\hat{b} > 0$. Finally, evaluating (21) at arbitrary $(\tau, t), (\tau, t') \in T_{N,K}$ we find that

$$\forall (\tau, t), (\tau, t') \in T_{N,K} : \frac{\hat{\psi}_1(t')\hat{\psi}_3(\tau-t')}{\psi_1(t')\psi_3(\tau-t')} = \frac{\hat{\psi}_1(t)\hat{\psi}_3(\tau-t)}{\psi_1(t)\psi_3(\tau-t)}.$$

Therefore, $\psi_1(t)$ and $\hat{\psi}_1(t)f^{\tau-t}$ are proportional given τ , and necessarily $\hat{\psi}_1(t) = \hat{c}f^t\psi(t)$ for some constant $\hat{c} > 0$. \square

³⁹Assumption 6 implies that, for $t = 1, 2, \dots, K$, there exist $\tau_t, \tau'_t \in T_N(t)$ such that the numerator and the denominator in (20) are different. Clearly, Assumption 6 is not a necessary condition for this to be true. However, milder conditions are not very transparent as these typically depend on all parameters of the model.

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Tables and figures

Table 1: Estimation results duration model

	females		males	
unobserved heterogeneity (G)				
μ_1	0.277	(0.005)	0.329	(0.004)
γ_2	1.295	(0.079)	1.192	(0.046)
γ_3	2.517	(0.429)	1.796	(0.231)
γ_4	7.221	(2.236)	3.287	(0.983)
γ_5	24.906	(11.533)	6.055	(4.128)
duration dependence ($\ln \psi_1(t)$)				
quarter 0	0		0	
quarter 1	0.160	(0.040)	0.077	(0.030)
quarter 2	0.043	(0.075)	-0.020	(0.045)
quarter 3	0.071	(0.079)	-0.066	(0.051)
quarter 4	0.132	(0.091)	-0.001	(0.056)
contemporaneous trend and cycle ($\alpha_2(y)$)				
trend ($\Delta(\alpha_2 - \alpha_2^\perp)$)	-0.020	(0.002)	-0.024	(0.001)
cycle ($\alpha_2^\perp(y)$):				
1982.IV–1983.III	0.138	(0.019)	0.007	(0.014)
1983.IV–1984.III	-0.026	(0.014)	-0.105	(0.011)
1984.IV–1985.III	-0.035	(0.013)	-0.010	(0.009)
1985.IV–1986.III	-0.106	(0.015)	-0.027	(0.011)
1986.IV–1987.III	-0.044	(0.019)	0.035	(0.013)
1987.IV–1988.III	-0.011	(0.019)	0.072	(0.013)
1988.IV–1989.III	0.031	(0.017)	0.093	(0.013)
1989.IV–1990.III	0.045	(0.016)	0.085	(0.013)
1990.IV–1991.III	-0.037	(0.015)	-0.017	(0.012)
1991.IV–1992.III	0.021	(0.014)	-0.025	(0.011)
1992.IV–1993.III	0.006	(0.015)	-0.093	(0.013)
1993.IV–1994.III	0.017	(0.020)	-0.014	(0.017)

(table continued on next page)

Table 1: (Continued)

	females		males	
contemporaneous season ($\omega_2(s)$)				
quarter I	0.083	(0.017)	0.140	(0.012)
quarter II	0.026	(0.015)	0.046	(0.012)
quarter III	0.054	(0.017)	0.021	(0.013)
quarter IV	0		0	
cycle composition inflow ($\alpha_3(y) = \alpha_3^\perp(y)$)				
1982.IV–1983.III	-0.072	(0.018)	0.016	(0.014)
1983.IV–1984.III	-0.003	(0.013)	0.011	(0.009)
1984.IV–1985.III	-0.005	(0.011)	-0.017	(0.008)
1985.IV–1986.III	-0.010	(0.014)	-0.028	(0.009)
1986.IV–1987.III	0.040	(0.016)	-0.001	(0.011)
1987.IV–1988.III	0.030	(0.016)	-0.003	(0.011)
1988.IV–1989.III	0.037	(0.016)	0.010	(0.012)
1989.IV–1990.III	0.052	(0.015)	0.012	(0.012)
1990.IV–1991.III	0.050	(0.013)	0.021	(0.011)
1991.IV–1992.III	0.000	(0.012)	-0.019	(0.010)
1992.IV–1993.III	-0.057	(0.013)	-0.006	(0.011)
1993.IV–1994.III	-0.063	(0.019)	0.003	(0.015)
season composition inflow ($\omega_3(s)$)				
quarter I	-0.072	(0.006)	-0.076	(0.004)
quarter II	-0.060	(0.006)	-0.084	(0.006)
quarter III	0.021	(0.005)	0.001	(0.004)
quarter IV	0		0	
measurement error				
σ	0.017	(0.001)	0.015	(0.001)
ρ	0.323	(0.086)	0.326	(0.086)
$d_{\geq '87}$	0.159	(0.019)	0.133	(0.013)
$N + 1(K + 1)$	48(5)		48(5)	
$\ln \mathcal{L}$	790.034		813.062	

(table continued on next page)

Table 1: (Continued)

	females	males
Wald statistics		
duration dependence (4)	31.8	31.6
contemporaneous cycle (10)	140.1	212.2
contemporaneous season (3)	24.4	185.0
cycle composition inflow (10)	54.8	30.3
season composition inflow (3)	241.6	484.3
Pseudo-R^2		
R_0^2	0.80	0.92
R_1^2	0.84	0.93
R_2^2	0.74	0.91
R_3^2	0.39	0.79
R_4^2	0.46	0.47
Durbin-Watson statistics		
DW_0	1.70	1.57
DW_1	2.66	2.34
DW_2	2.00	1.83
DW_3	1.97	2.09
DW_4	1.27	1.51
Moment-inequality statistics (t-values)		
$\gamma_2 - 1$	3.74	4.16
$\gamma_3 - \gamma_2^2$	3.03	2.78
$\gamma_2\gamma_4 - \gamma_3^2 - \gamma_4 - \gamma_2^3 + 2\gamma_2\gamma_3$	1.72	-0.24
$\gamma_3\gamma_5 - \gamma_4^2 - \gamma_2^2\gamma_5 - \gamma_3^3 + 2\gamma_2\gamma_3\gamma_4$	-0.14	-1.05

Explanatory note: Standard errors are in parentheses. The Wald test statistics concern the null hypotheses $\ln \psi_1(1) = \ln \psi_1(2) = \ln \psi_1(3) = \ln \psi_1(4) = 0$, and $\alpha_k^\perp(1982.IV-1983.III) = \dots = \alpha_k^\perp(1993.IV-1994.III) = 0$ and $\omega_k(I) = \omega_k(II) = \omega_k(III) = 0$ for $k = 1, 2$. Degrees of freedom are given in parentheses. R_i^2 , $i = 0, \dots, 4$, are pseudo- R^2 statistics for the $i + 1$ -th equation ($\tilde{\theta}(i|\tau)$). DW_i , $i = 0, \dots, 4$, are Durbin-Watson statistics for the $i + 1$ -th equation ($\tilde{\theta}(i|\tau)$). The moment-inequality tests are explained in Subsection 2.5; t -values are reported here.

Table 2: Estimation results incidence model

	females	males
constant (ν_1)	359.437 (9.493)	455.327 (7.328)
trend and cycle incidence ($\alpha_4(y)$)		
trend ($\Delta(\alpha_4 - \alpha_4^\perp)$)	0.027 (0.004)	0.018 (0.003)
cycle ($\alpha_4^\perp(y)$):		
1982.IV–1983.III	-0.017 (0.015)	-0.020 (0.009)
1983.IV–1984.III	0.003 (0.016)	0.051 (0.010)
1984.IV–1985.III	-0.044 (0.019)	-0.012 (0.011)
1985.IV–1986.III	-0.055 (0.021)	-0.042 (0.013)
1986.IV–1987.III	0.017 (0.019)	0.047 (0.012)
1987.IV–1988.III	0.062 (0.023)	0.041 (0.014)
1988.IV–1989.III	0.038 (0.021)	-0.025 (0.013)
1989.IV–1990.III	0.022 (0.019)	-0.082 (0.011)
1990.IV–1991.III	0.038 (0.017)	-0.028 (0.010)
1991.IV–1992.III	0.012 (0.016)	-0.001 (0.010)
1992.IV–1993.III	0.020 (0.017)	0.078 (0.010)
1993.IV–1994.III	-0.028 (0.018)	0.019 (0.011)
1994.IV(–1995.III)	-0.067 (0.028)	-0.028 (0.017)
season incidence ($\omega_4(s)$)		
quarter I	-0.172 (0.014)	-0.202 (0.009)
quarter II	-0.200 (0.014)	-0.331 (0.009)
quarter III	0.207 (0.014)	-0.020 (0.009)
quarter IV	0	0
measurement error		
σ	0.033 (0.003)	0.020 (0.002)
$d_{\geq '87}$	0.026 (0.040)	-0.039 (0.024)
$N + 2$	49	49
$\ln \mathcal{L}$	142.114	166.427
Wald statistics		
cycle incidence (11)	18.6	226.1
season incidence (3)	1137.1	2095.3
Pseudo-R^2		
R^2	0.97	0.98
Durbin-Watson statistic		
DW	2.09	2.27

Explanatory note: Standard errors are in parentheses. The Wald test statistics concern the null hypotheses $\alpha_4^\perp(1982.IV-1983.III) = \dots = \alpha_4^\perp(1994.IV-1995.III) = 0$ and $\omega_4(I) = \omega_4(II) = \omega_4(III) = 0$. Degrees of freedom are given in parentheses. DW is the Durbin-Watson statistic and R^2 the pseudo- R^2 for the incidence equation.

Figure 1: Unemployment ($\tilde{U}(\tau)$; deseasonalized)

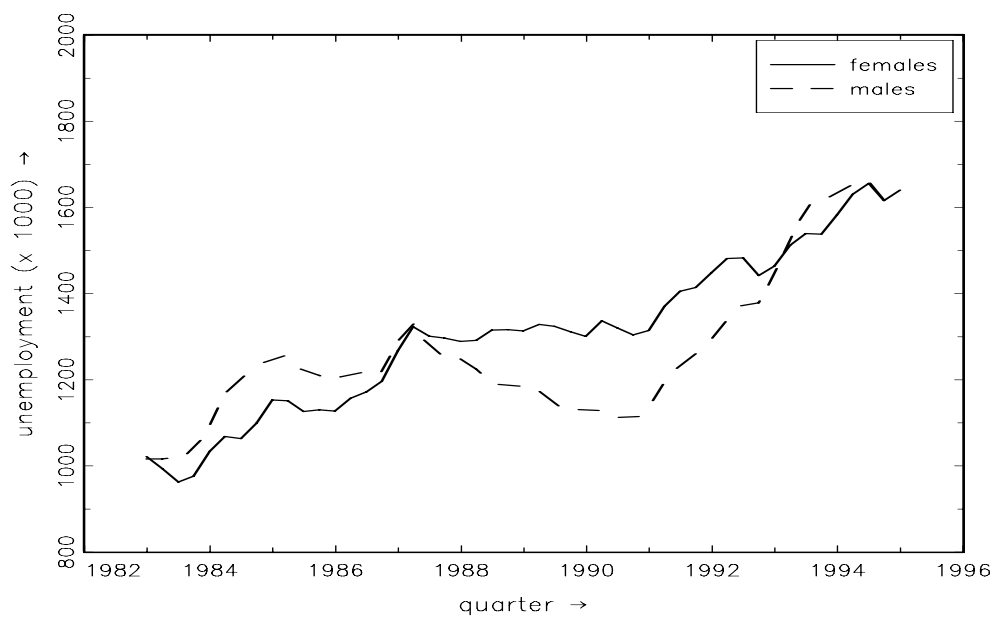


Figure 2: Business cycle indicators (detrended and deseasonalized; mean norm. to 0)

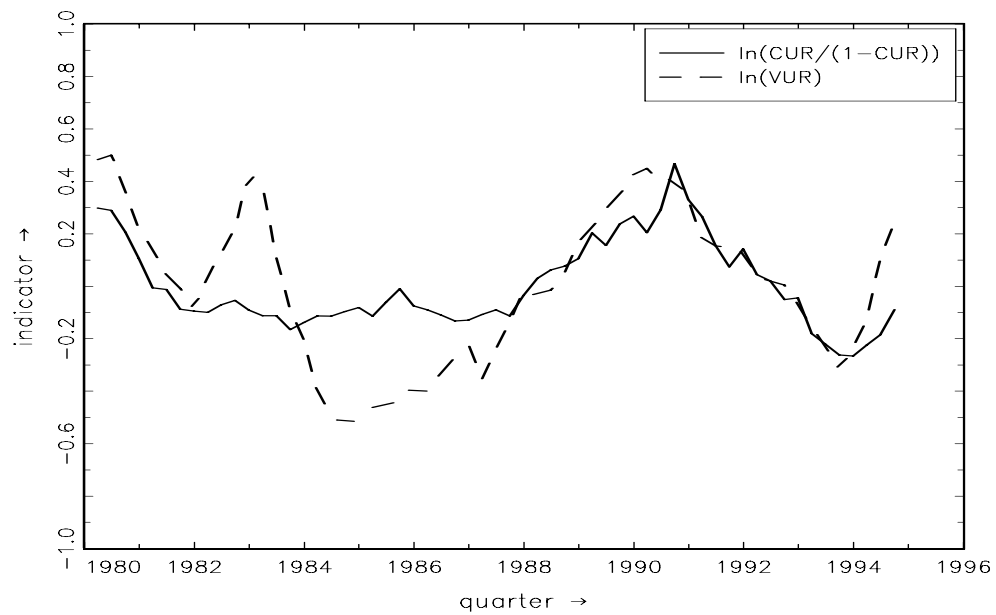


Figure 3: Unemployment ($\tilde{U}(\tau)$; deseasonalized and detrended)

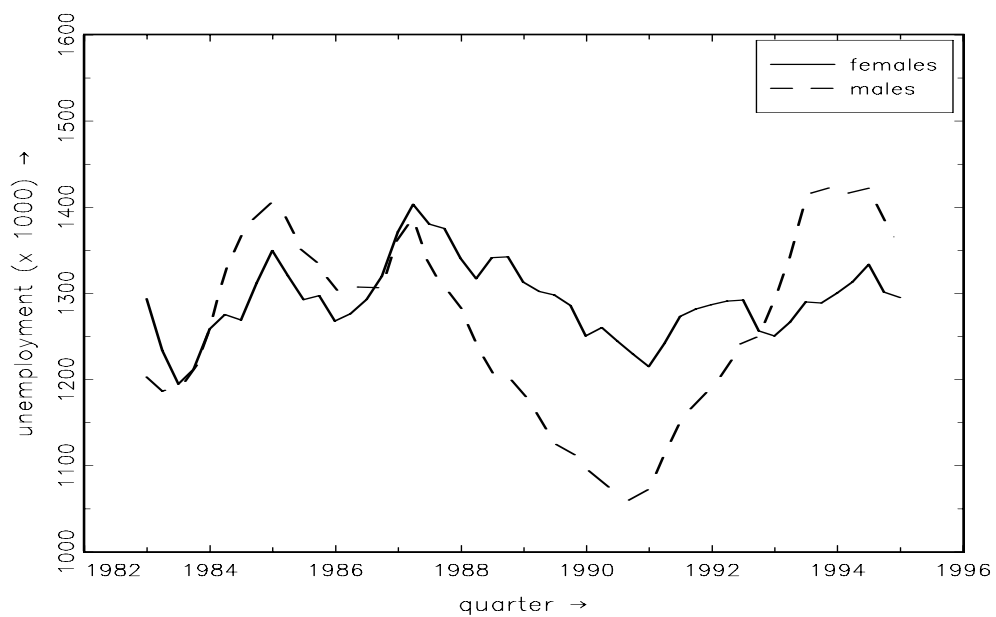


Figure 4: Inflow into unemployment ($\tilde{U}(0|\tau)$; deseasonalized)

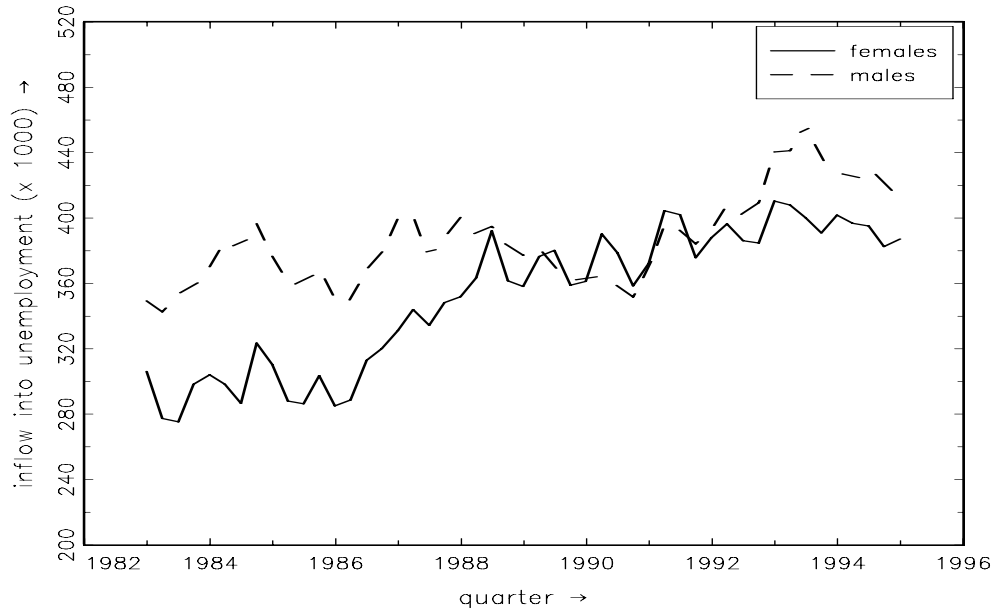


Figure 5: Outflow from unemployment ($\tilde{U}(0|\tau + 1) + \tilde{U}(\tau) - \tilde{U}(\tau + 1)$; deseasonalized)

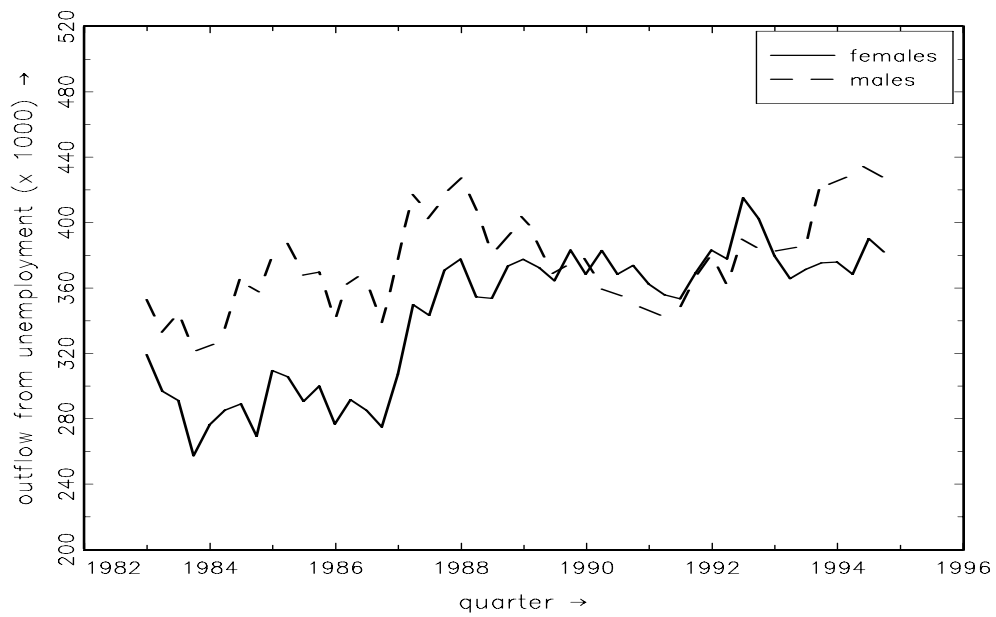


Figure 6: Quarterly exit probabilities by duration class ($\tilde{\theta}(t|\tau)$); females (top) and males (bottom)

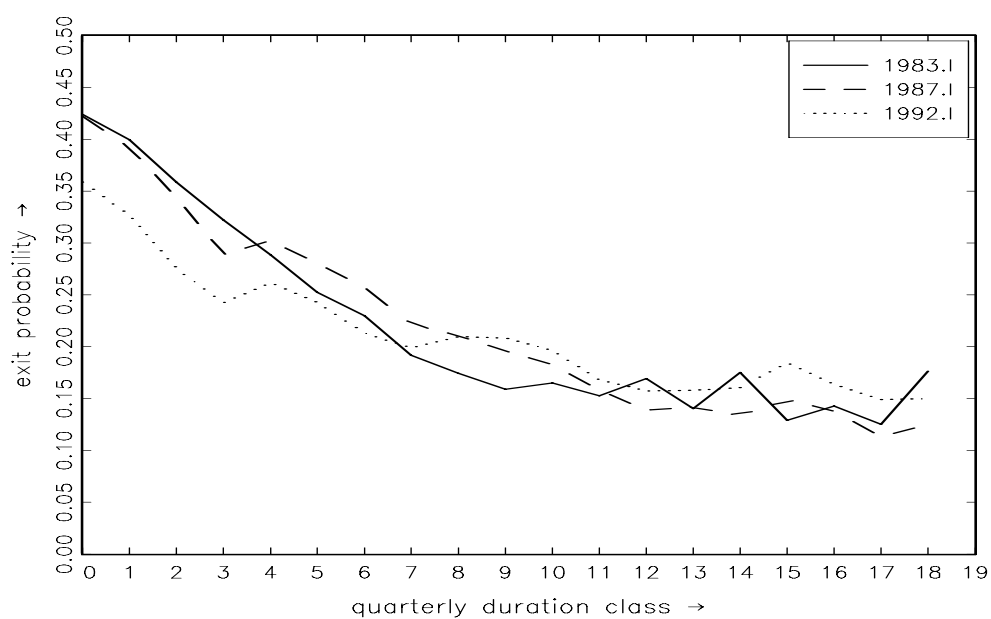
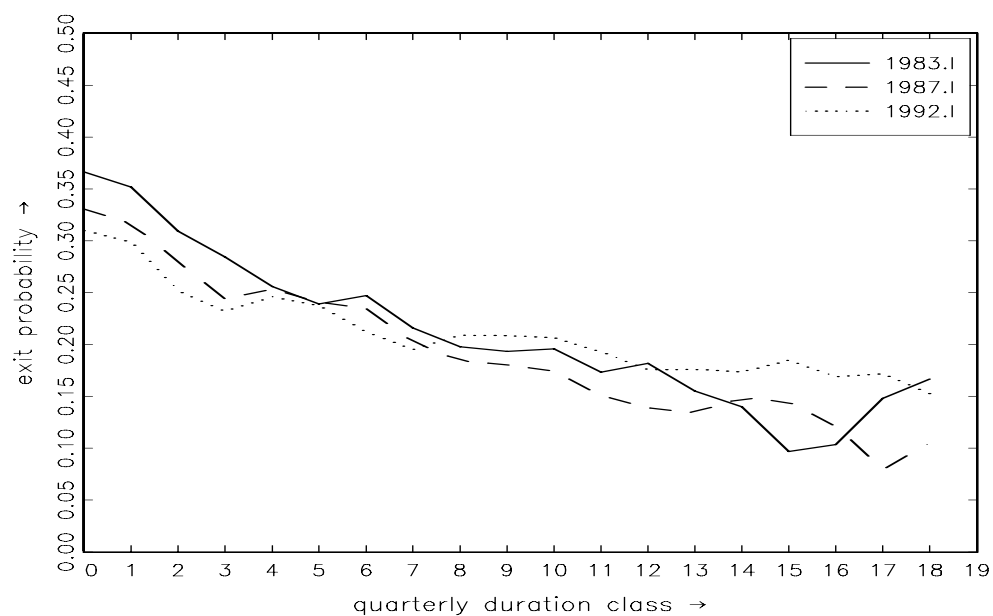


Figure 7: Individual duration dependence ($\ln \psi_1(t)$)

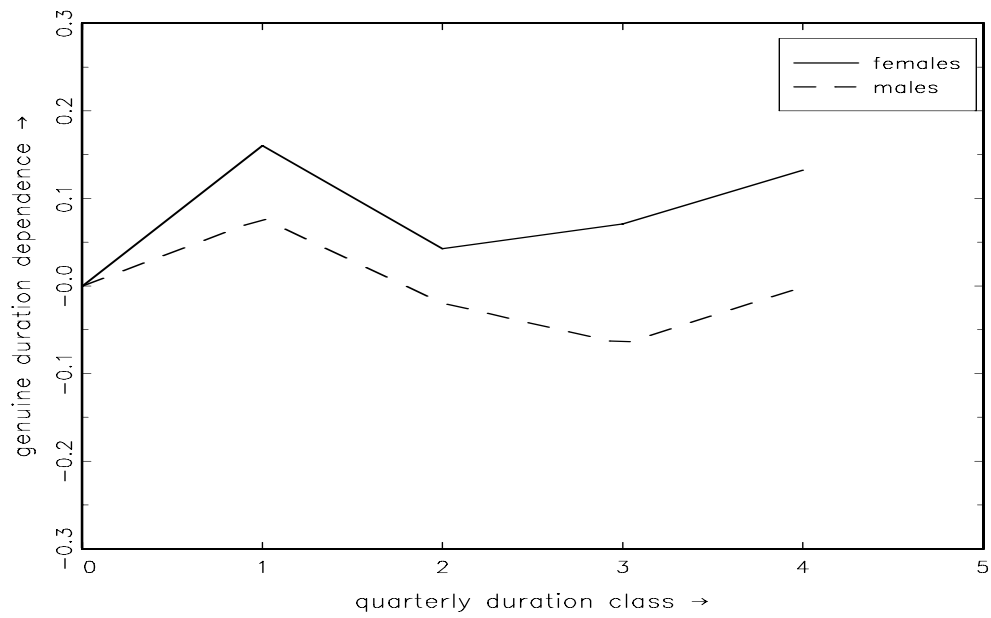


Figure 8: Effect of unobserved heterogeneity (log interaction term); females (top) and males (bottom)

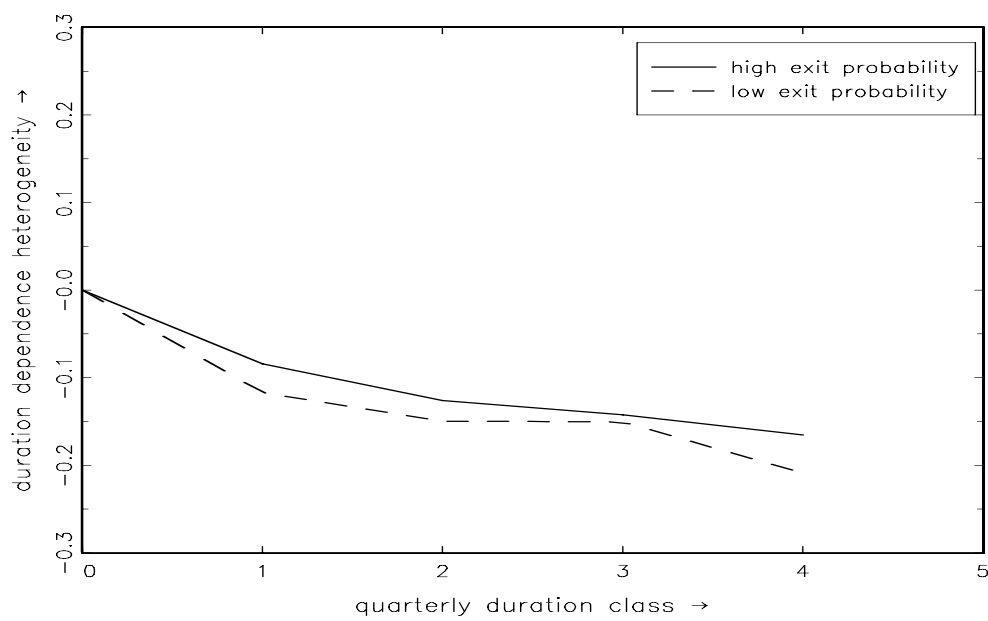
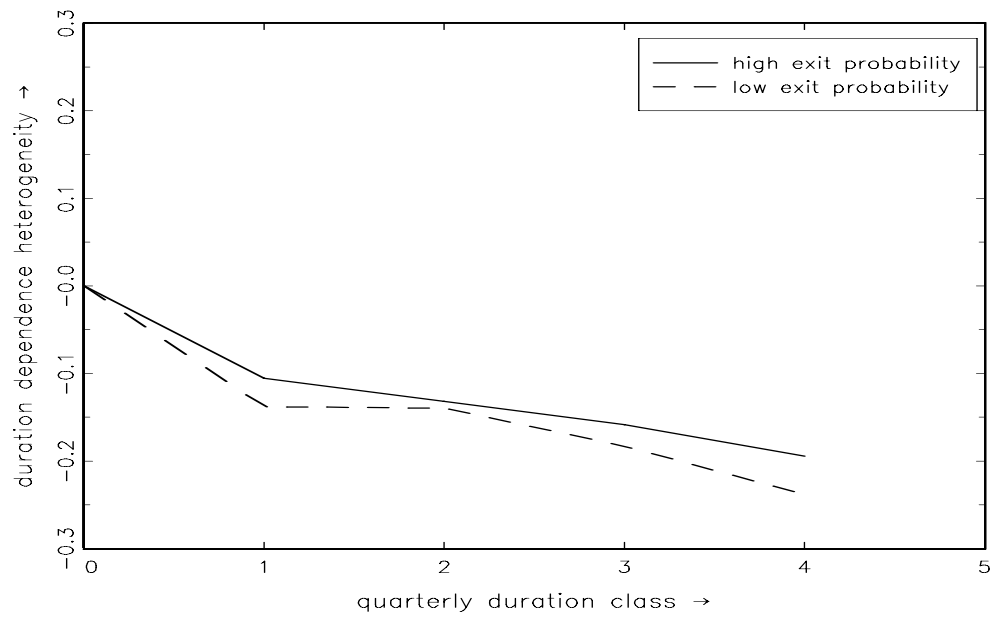


Figure 9: Contemporaneous cycle exit probabilities ($\alpha_2^\perp(y(\tau))$)

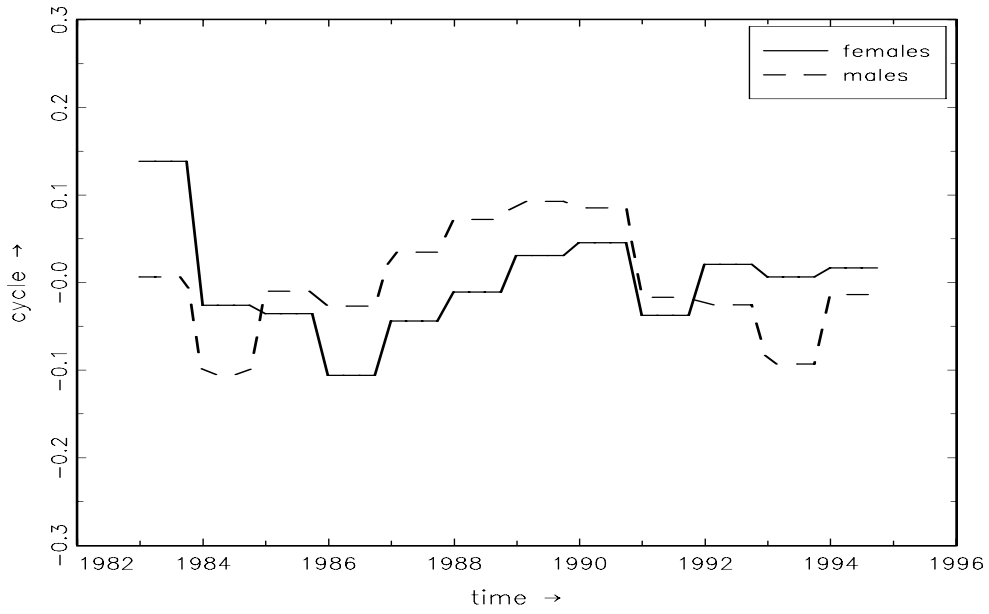


Figure 10: Cycle composition inflow ($\alpha_3^\perp(y(\tau)) = \alpha_3(y(\tau))$)

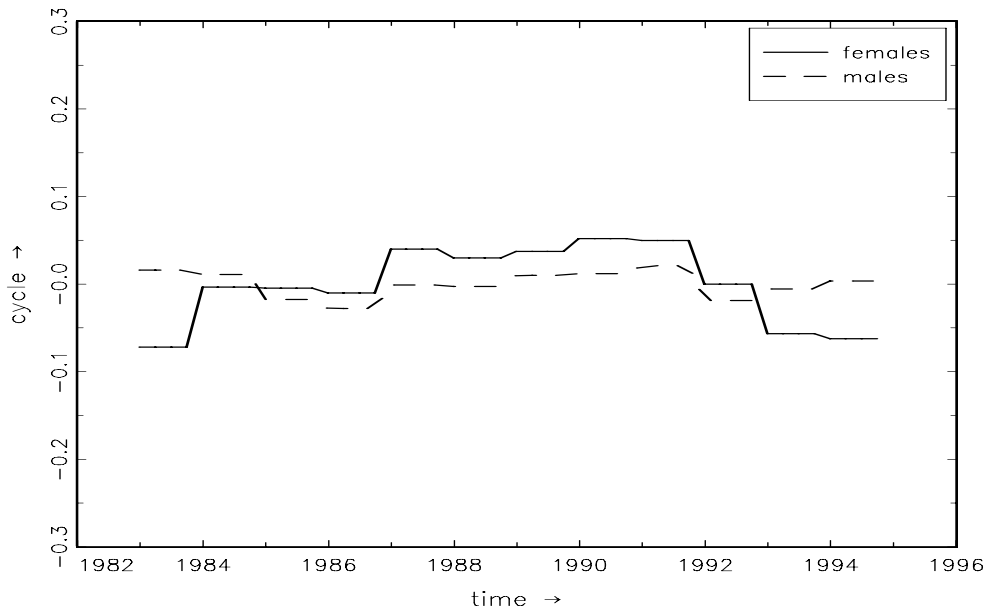


Figure 11: Cycle incidence ($\alpha_4^\perp(y(\tau))$)

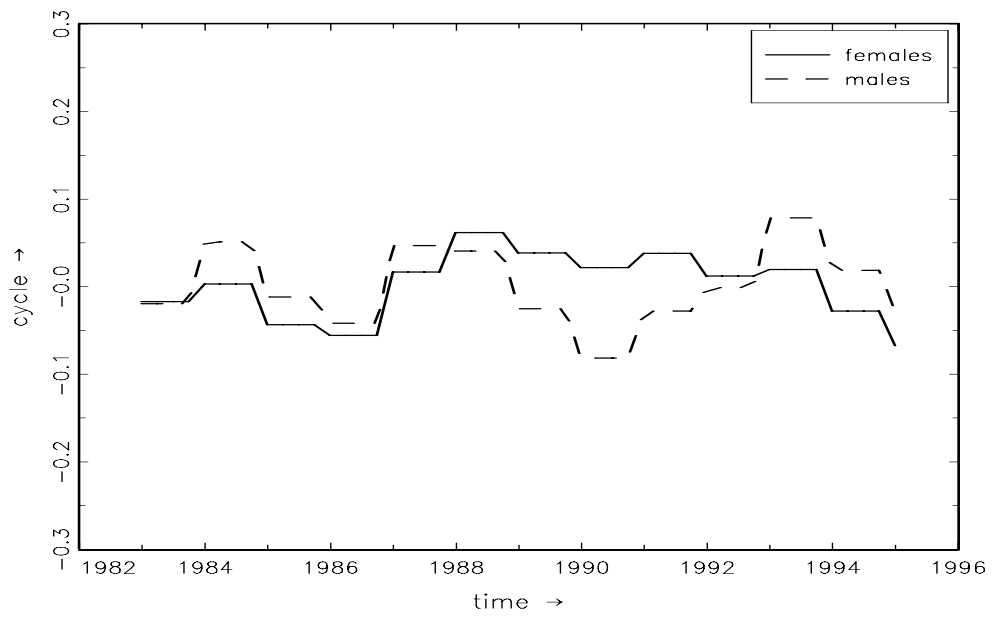


Figure 12: Unemployed in fifth quarter ($U(4|\tau)$); females (top) and males (bottom)

